

Crustal Thickness Modeling

Assumptions employed:

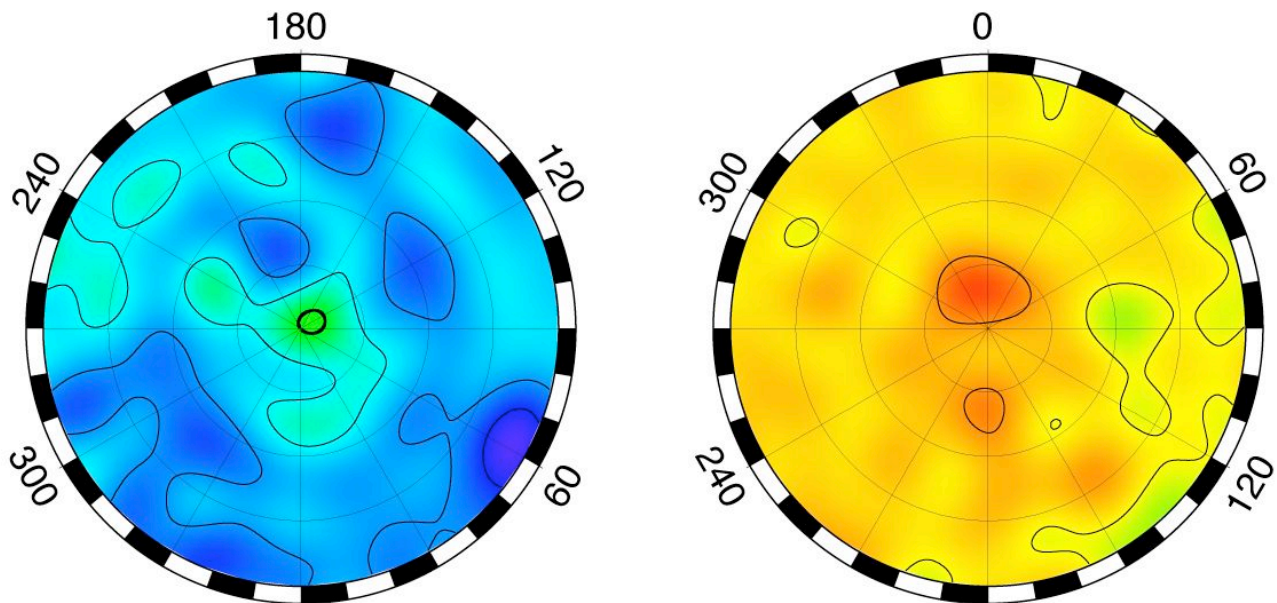
1. Mars is composed solely of a constant density crust, mantle and core.
2. The average thickness of the crust is either set to be 50 km, or the minimum thickness of the crust is set to ~ 0 km.

Modeling Procedure:

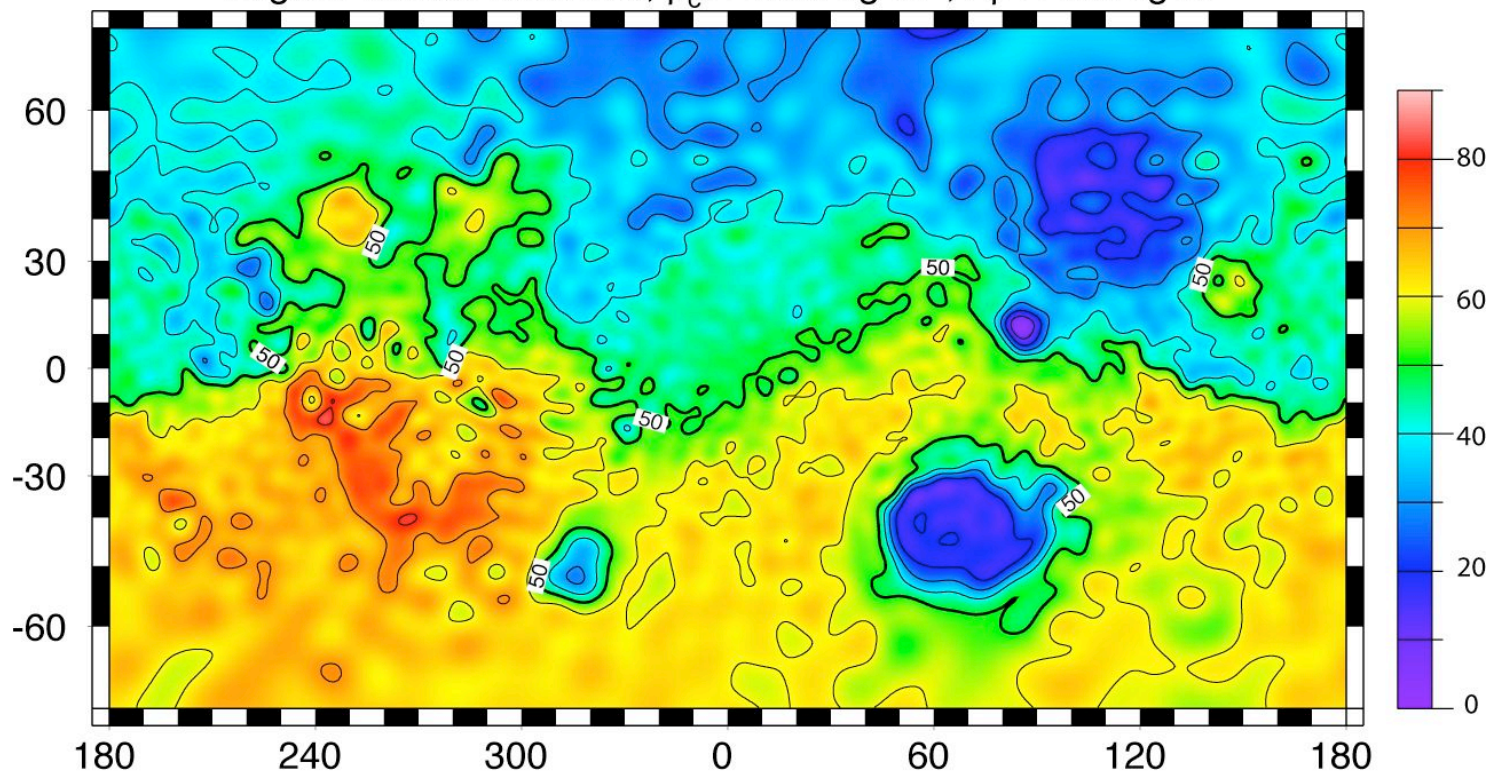
1. Compute Bouguer correction (gravity due to surface topography)
2. Compute Bouguer Anomaly (observed gravity - Bouguer correction)
3. Interpret Bouguer Anomaly as relief along the crust-mantle interface

Shortcomings:

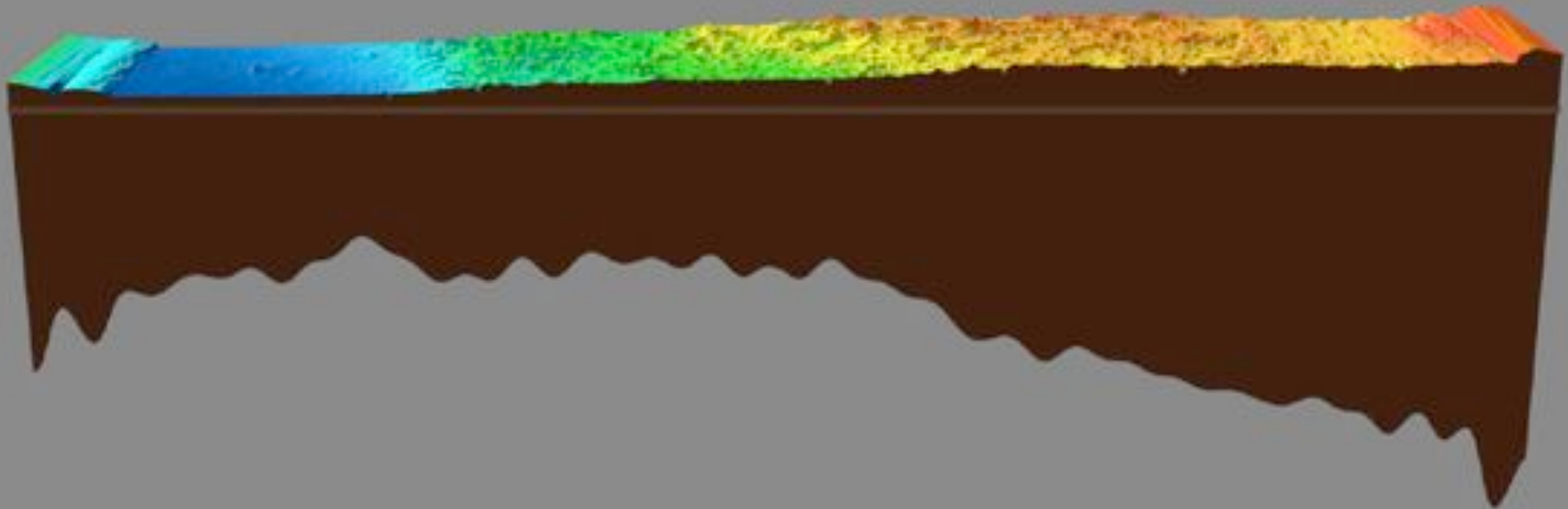
1. The densities of the major volcanoes are probably greater than average.
2. Does not take into account volcanic intrusions beneath volcanic edifices.



Deg. 50 crustal thickness, $\rho_c = 3000 \text{ kg m}^{-3}$, $\Delta\rho = 600 \text{ kg m}^{-3}$



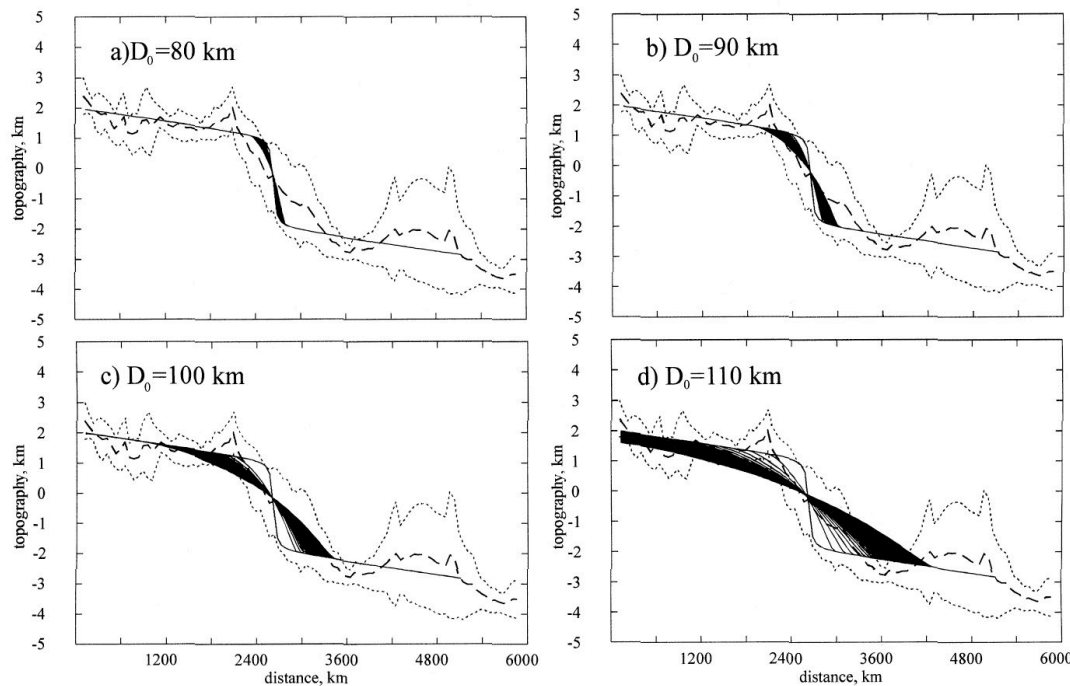
Pole-to-Pole Crustal Thickness Profile



- This crustal thickness inversion assumed that the density of the crust was constant.
- If the density of the crust in the northern hemisphere is greater than that of the highlands (like on Earth), then it is possible that the pole-to-pole crustal thickness variations could be drastically reduced.

Viscous Relaxation of Crustal Topography

Highland Dichotomy Boundary



1. Temperature increases with depth in the crust.
2. Viscous flow of crustal materials depends on the viscosity, which depends upon temperature.
3. The thicker the crust, the faster that viscous relaxation will occur.
4. The average thickness of the crust must be less than ~ 110 km.

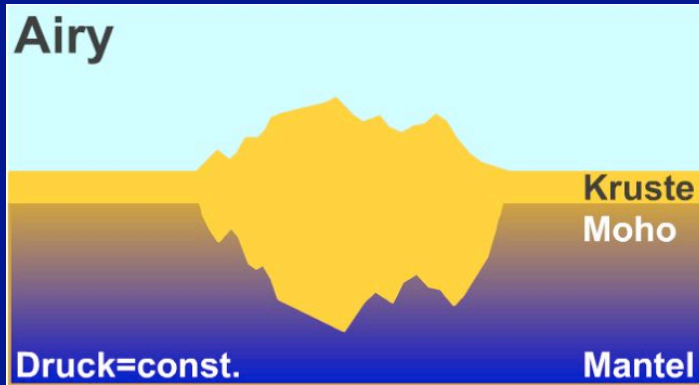
Nimmo and Stevenson (2001)

The Average Thickness of the Crust

Method	Average Crustal Thickness	Reference
Crustal Thickness modeling (assuming minimum crustal thickness is 3 km)	> 32 km	<i>Wieczorek and Zuber</i> (submitted)
Viscous Relaxation	< 115 km	<i>Nimmo and Stevenson</i> (2001)
Geoid-to-Topography Ratios	57 ± 24 km	<i>Wieczorek and Zuber</i> (submitted)
Spectral Admittances		
Hellas	38 - 62 km	<i>McGovern et al.</i> (2002)
Noachis Terra	8 - 62 km	

Planet	Volume % Crust (silicate portion of planet)
Earth	~ 1 %
Mars	3 - 6 %
Moon	7 - 9 %

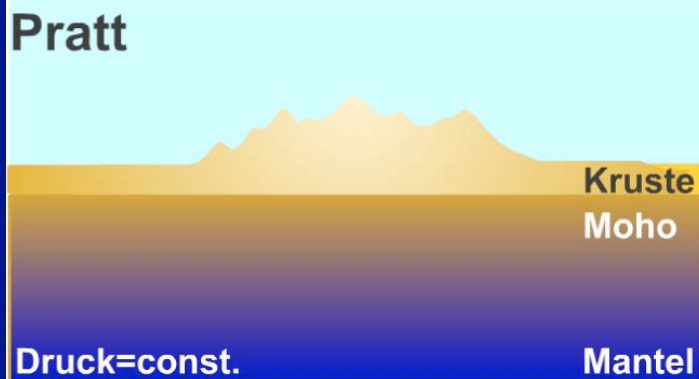
How is Surface Topography Supported (or Compensated)?



Airy Compensation

1. Rigid crustal columns “float” in the mantle.
2. Surface Topography has a crustal “root”.

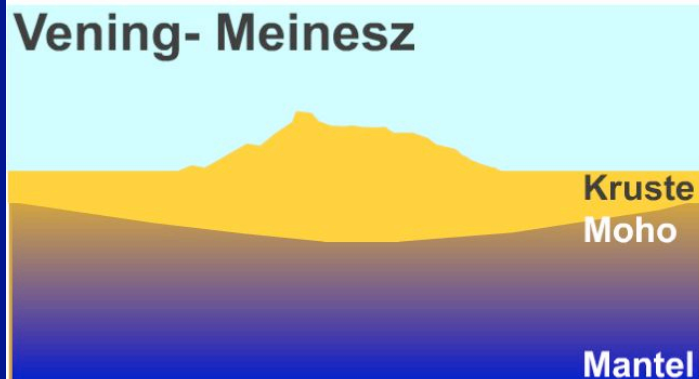
$$h \rho_c = (\rho_m - \rho_c) b$$



Pratt Compensation

1. The pressure at the base of the crust is constant.
2. Where the surface topography is high, the underlying crustal density is low.

$$H_0 \rho_0 = (H_0 + h)\rho$$



Flexure

1. Elastic stresses in the crust can partially support a load on the crust.
2. When the “elastic thickness” is zero, this is analogous to Airy compensation.

Flexure of an Elastic Plate

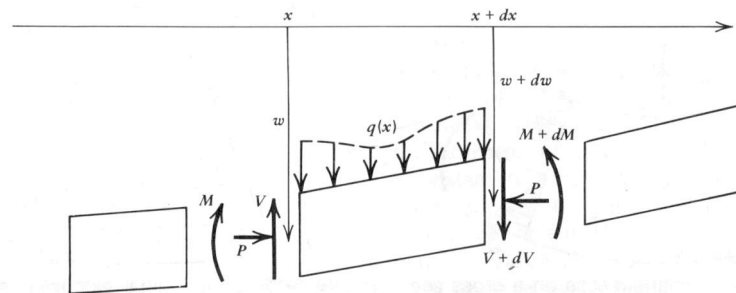
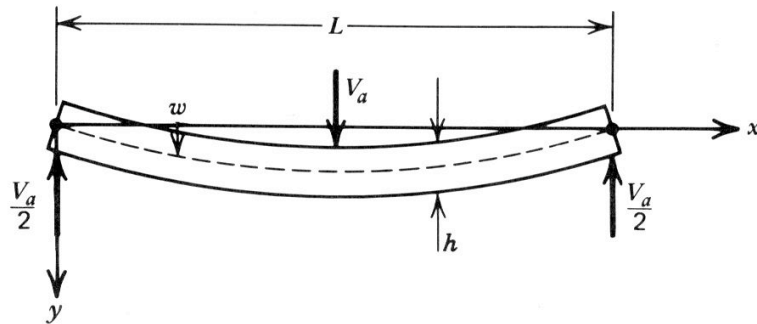


Figure 3-10 Forces and torques on a small section of a deflecting plate.

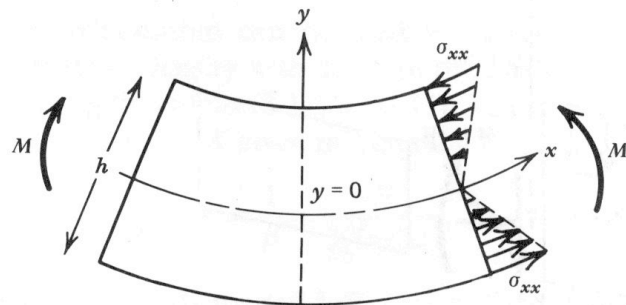


Figure 3-11 The normal stresses on a cross section of a thin curved elastic plate.

Force balance includes:

Applied load, q

Hydrostatic Restoring Force, $\Delta \rho g w$

In-plane forces, P

Shear forces, V

Bending moments, M

$$\frac{\partial^2 M(x)}{\partial x^2} - P(x) \frac{\partial^2 w(x)}{\partial x^2} = -q(x)$$

$$M = \int_{-h/2}^{h/2} y \sigma_{xx} dy$$

$$q = \rho_c g h + (\rho_m - \rho_c) g w$$

Rheology independent

$$\sigma_{xx} = \frac{E}{(1-\nu^2)} \epsilon_{xx} = -y \frac{E}{(1-\nu^2)} \frac{\partial^2 w}{\partial x^2}$$

$$M = -D \frac{\partial^2 w}{\partial x^2}$$

$$D = \frac{E h^3}{12(1+\nu^2)}$$

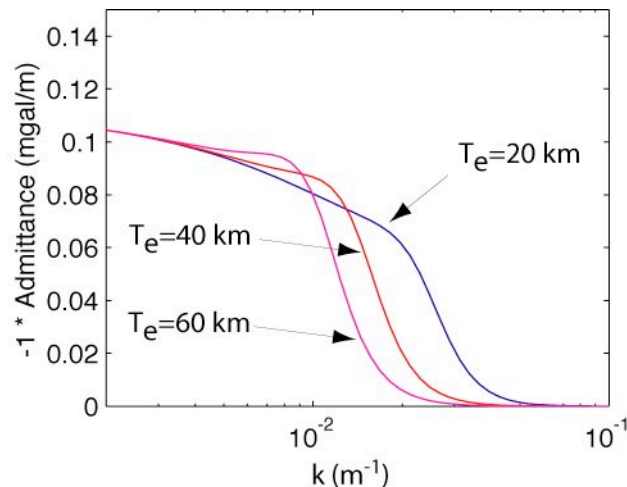
$$D \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} = q$$

Linear elasticity

Flexural Admittance Function

$$D \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} = q \quad \text{How do we solve this equation?}$$

Fourier transforming converts derivatives to multiplication by k ($2\pi/\lambda$).



Assuming no in-plane forces

$$w(k) = \frac{-\rho}{\Delta\rho + Dk^4 / g} h(k)$$

h = surface topography

In spherical coordinates, the flexure equation is

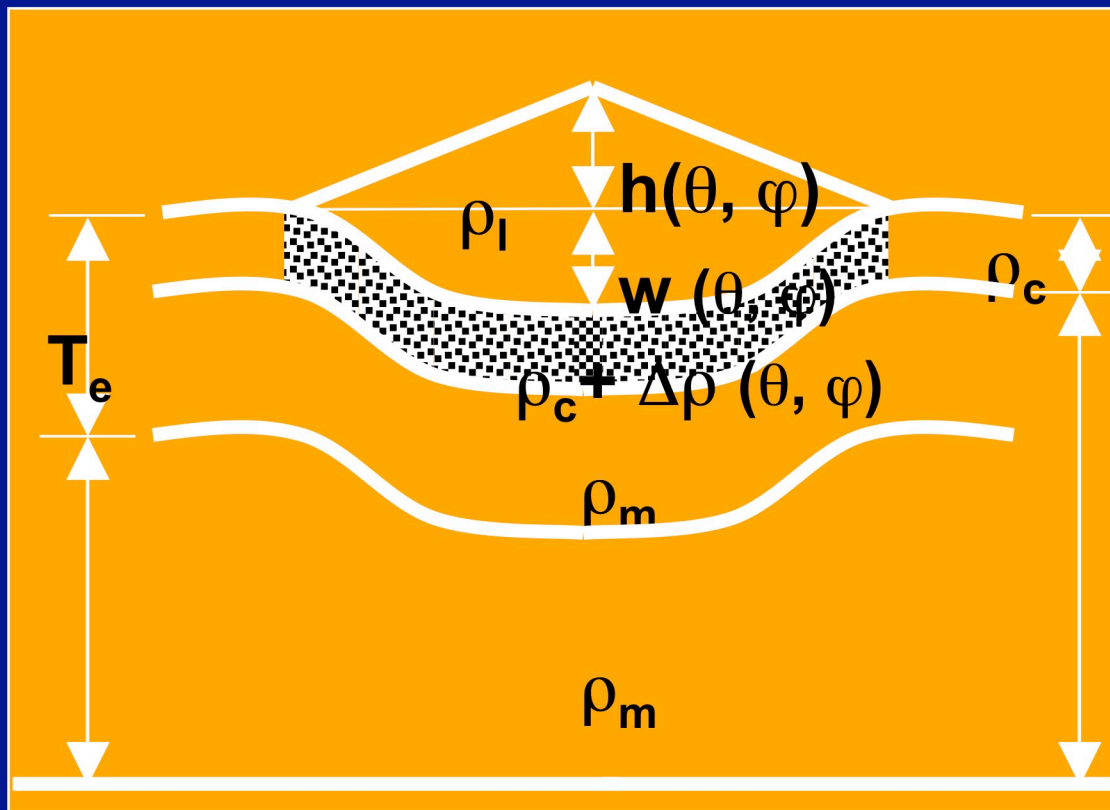
$$D\nabla^6 w + 4D\nabla^4 w + ET_e R^2 \nabla^2 w + 2ET_e R^2 w = R^4 (\nabla^2 + 1 - \nu) q$$

Expanding in spherical harmonics, ∇^2 is replaced by $-l(l+1)$.

Calcul de la charge

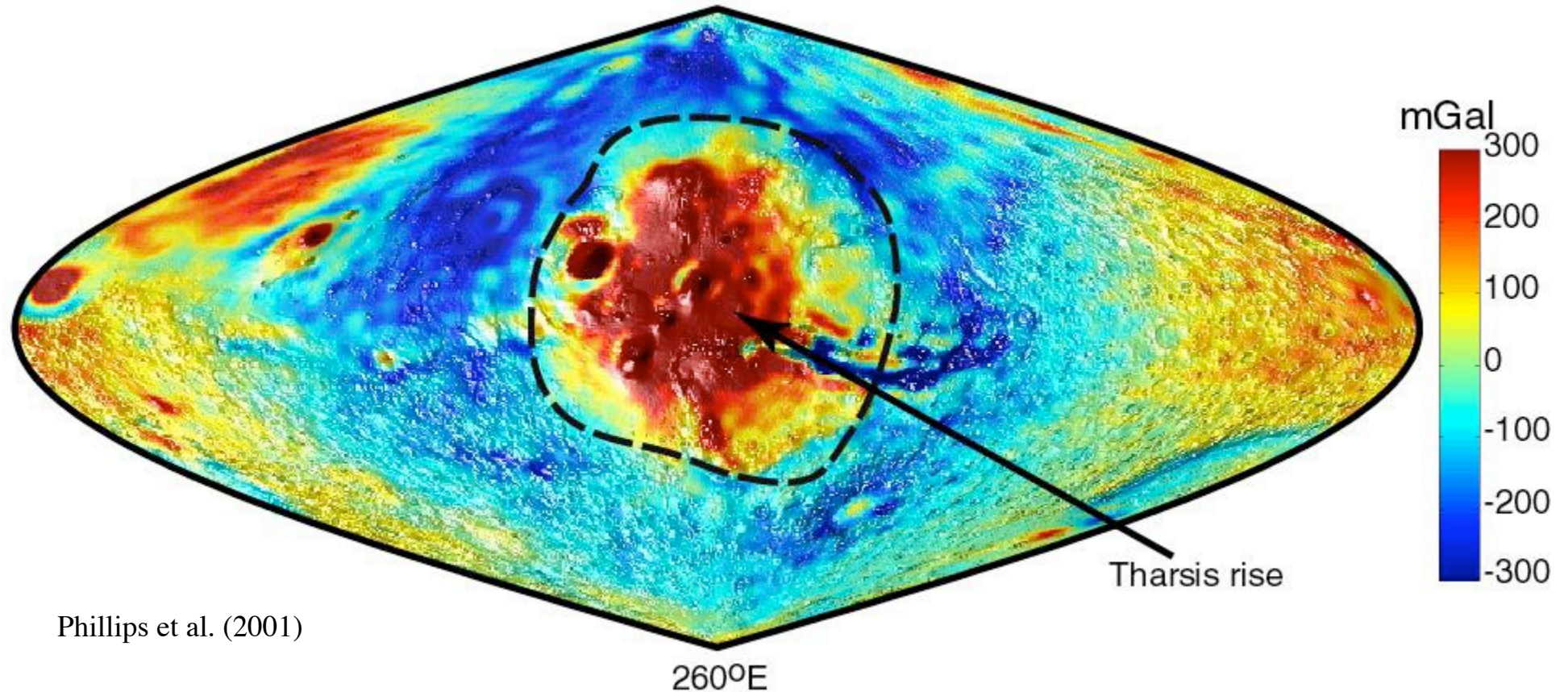
$$q = g[\rho_l h + (\rho_l - \rho_c)w + (\rho_c - \rho_m)w]$$

Si pas d'amincissement de croûte



Autres
relations
possibles

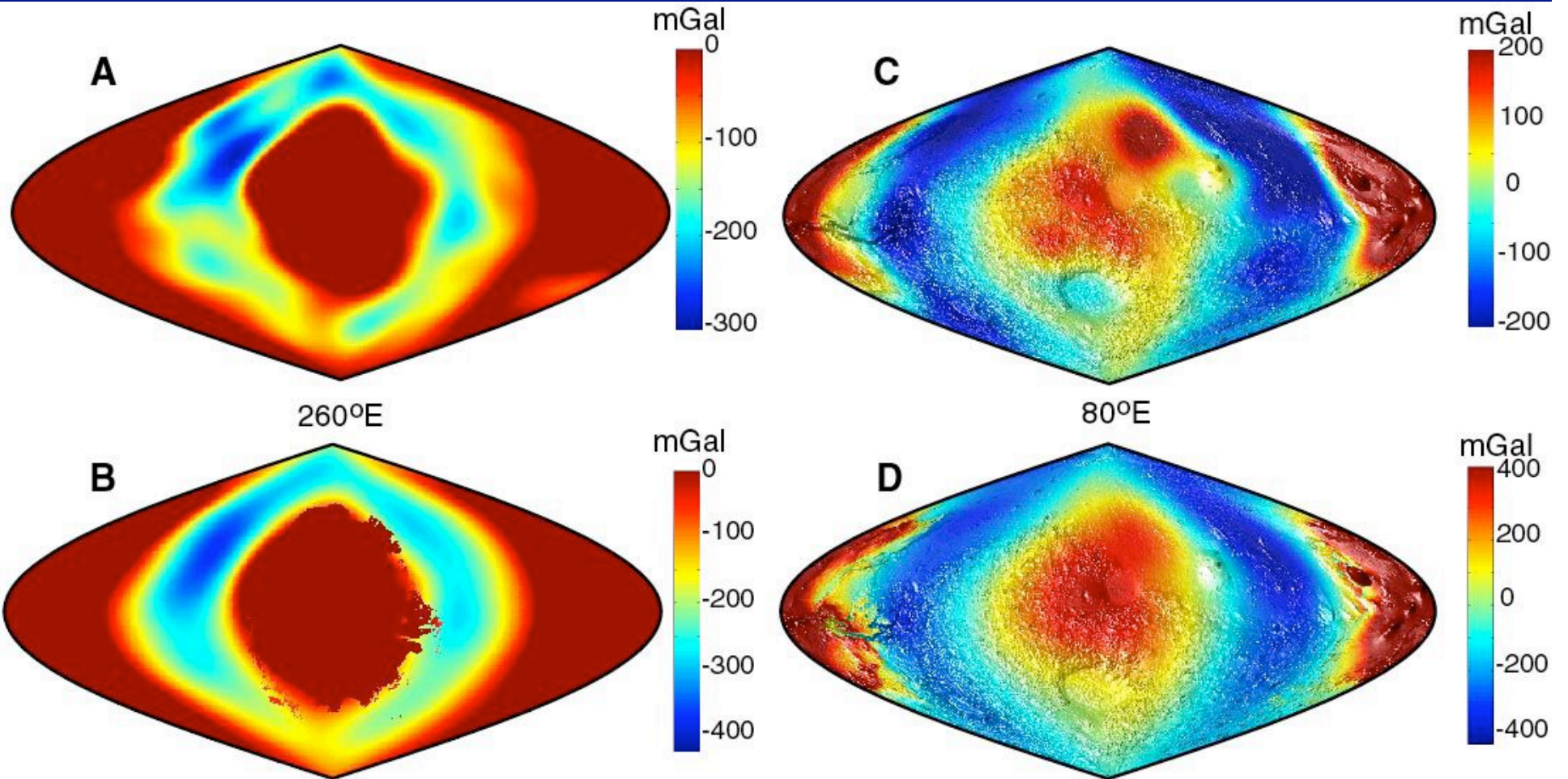
Gravity Field of Mars (Centered on Tharsis Rise)



Phillips et al. (2001)

How has the Tharsis volcanic load affected the gravity and topography of Mars?

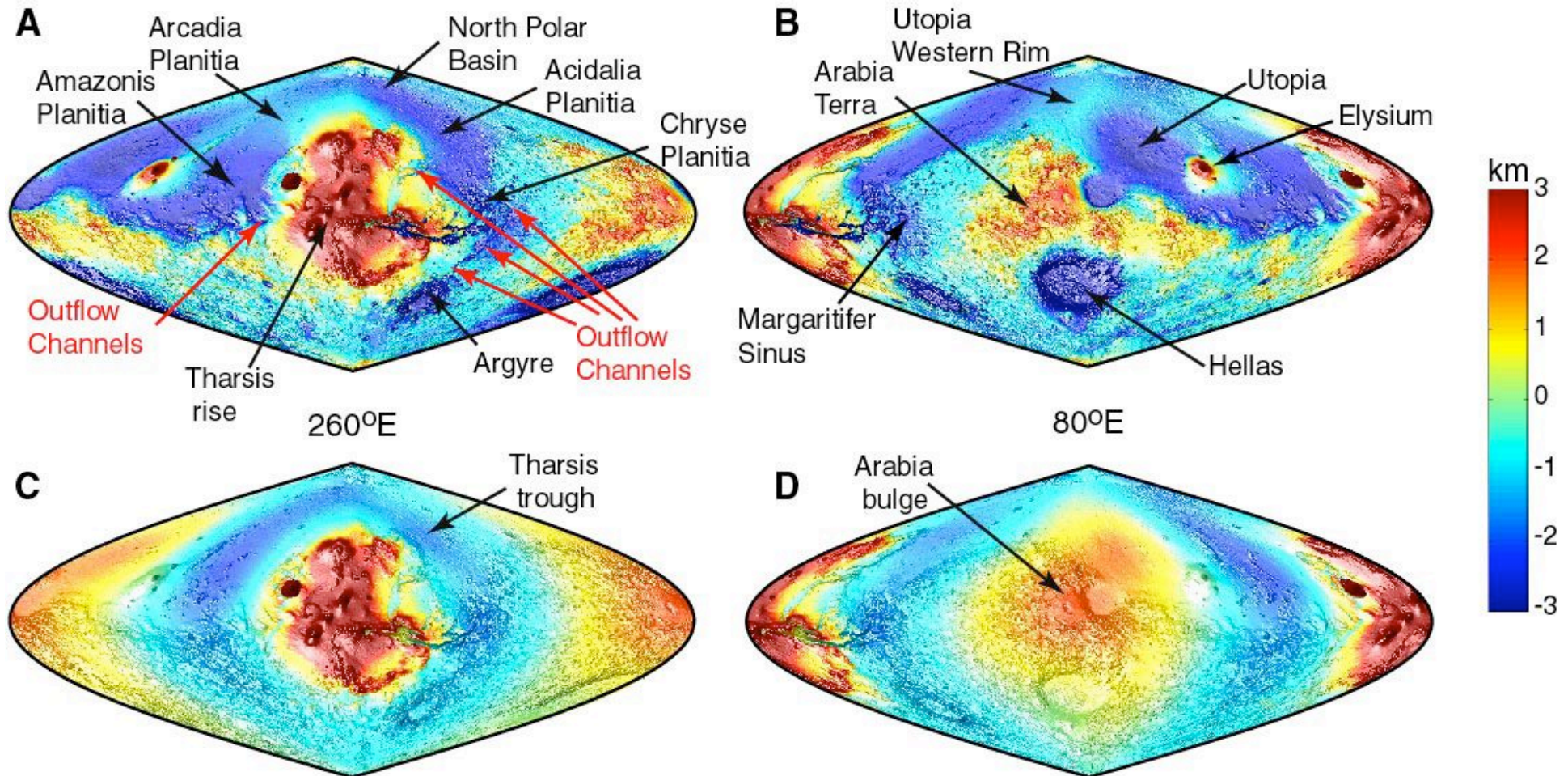
The long-wavelength gravity field ($l < 10$) of Mars can be explained as being the result of global flexure associated with the Tharsis Rise.



A. Modeled Tharsis Gravity
B. Observed Tharsis gravity

C. Modeled Antipodal Gravity
D. Observed Antipodal Gravity

Flexure associated with the Tharsis Rise forms a circum-Tharsis Trough, and antipodal buldge



Degree-1 topography has been removed in these images for clarity

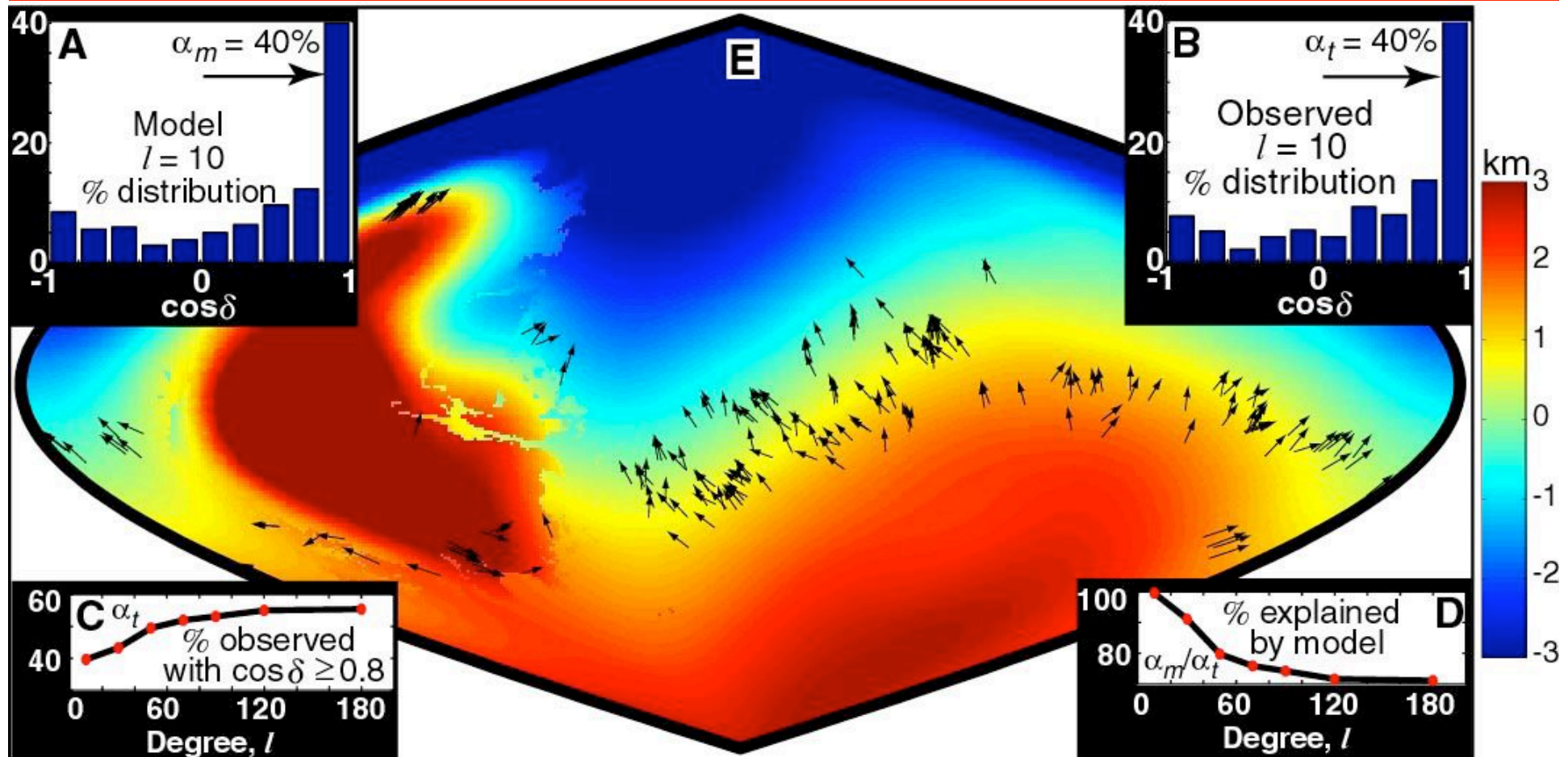
A. Observed Tharsis Topography

B. Modeled Tharsis Topography

C. Observed Antipodal Topography

D. Modeled Antipodal Topography

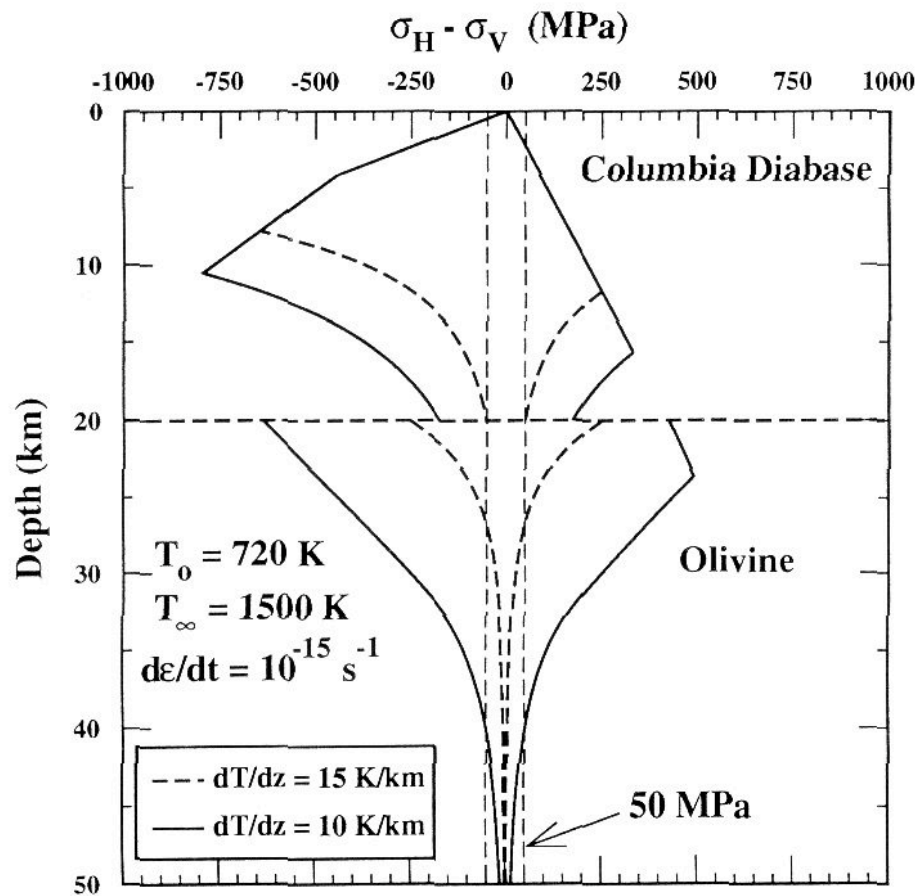
Noachian Valley Networks Directions are Controlled by the Topography caused by Tharsis Flexure



- The Tharsis load must have been emplaced before valley network formation!
- While the surface of Tharsis is young, the bulk of this region must have formed in the first ~500 Ma of Mars history

The Relationship Between T_e and Heat Flow

Lithospheric Yield Strength Envelope



The lithosphere is not perfectly elastic. When stresses exceed the “Yield Strength” the rock will deform either by faulting, or ductile creep.

The yield strength of the cold upper crust is described by Byerle’s Law.

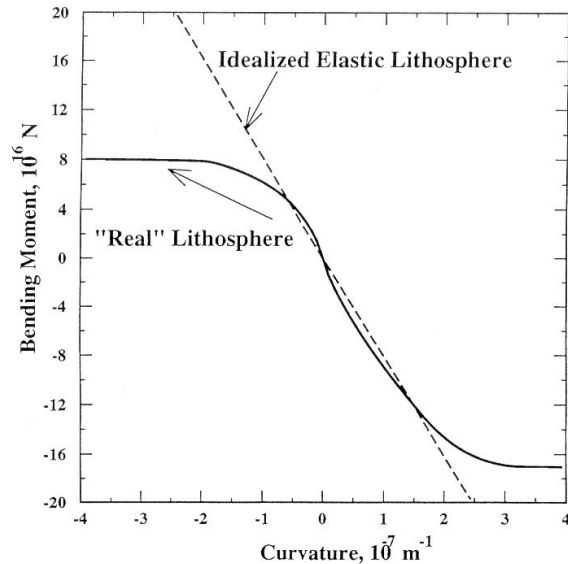
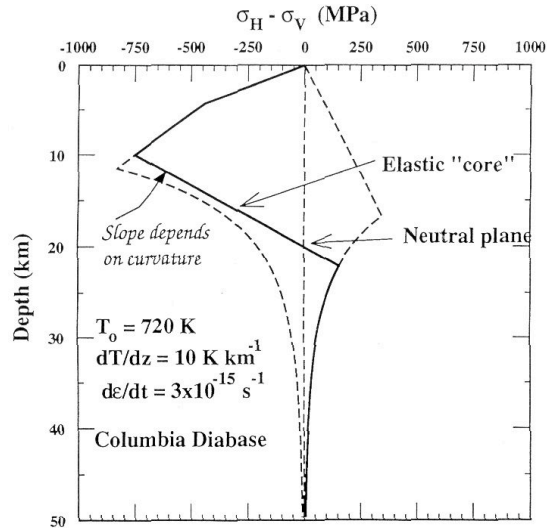
- The yield strength increases with increasing confining pressure.
- Rocks are stronger in compression than in tension.

The yield strength of the hotter lower crust and mantle is controlled by ductile flow.

- The yield strength is strongly temperature dependent
- The yield strength depends upon the strain rate.

The Strength of the lithosphere (effective elastic thickness) decreases with increasing temperature.

The Relationship Between T_e and Heat Flow



The bending moment for a plate with stresses limited by the yield strength envelope can still be calculated according to

$$M = \int_{-h/2}^{h/2} y \sigma_x dy$$

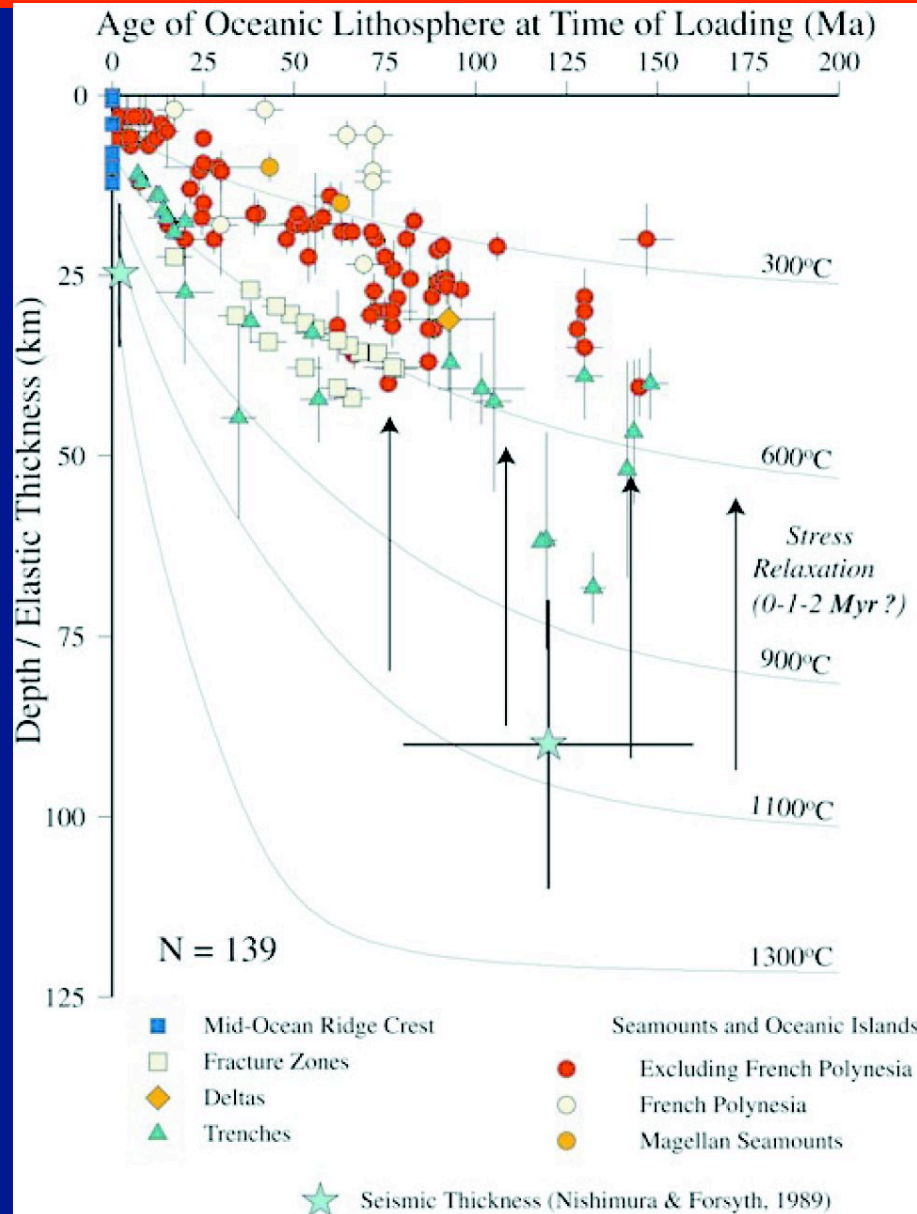
In order to match the observed flexural profile, the bending associated with a yield-strength envelope must be the same as that of a perfectly elastic plate

$$M = -D \frac{\partial^2 w}{\partial x^2}$$

How to calculate heat flux:

1. Calculate T_e , plate curvature, and bending moment.
2. For the observed curvature, vary the heat flux until the bending moment of a YSE equals that of the elastic plate.

When Was the Elastic Thickness Acquired?



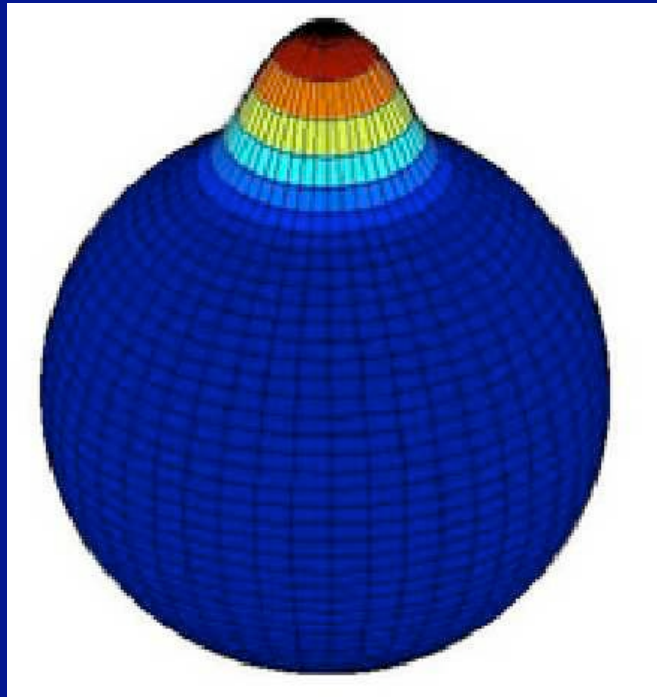
- In general, the temperature for a given region will decrease with time.
- Since the strength of the lithosphere increases with decreasing heat flux, the flexural profile at the time of loading will be "frozen in" as the plate cools.

The elastic thickness corresponds to the time of loading.

Example:

- The elastic thickness of the oceanic lithosphere closely approximates the depth of the 300-600 C isotherm at the time of loading.

3. Localized admittance calculation



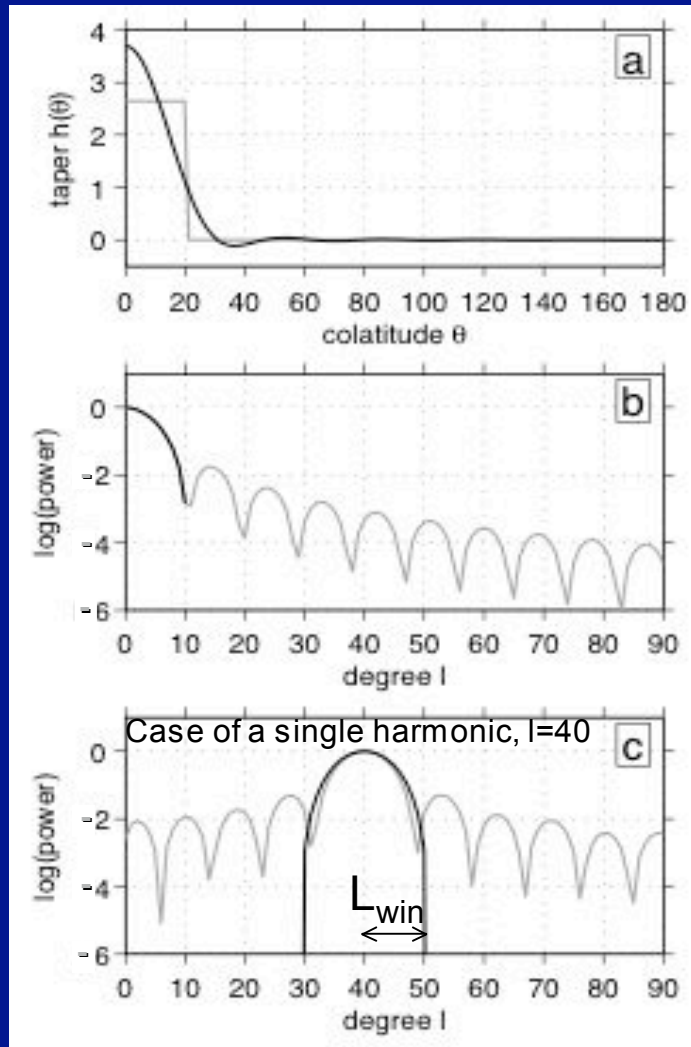
- Localized windows principle:
 - To multiply a window in space domain in order to isolate a signal.

$$\Psi(\Omega) = h(\Omega)W(\Omega)$$

$$\Gamma(\Omega) = g(\Omega)W(\Omega)$$

- The localized fields can then be studied in spectral domain using previous formulas.

3. Localized admittance calculation

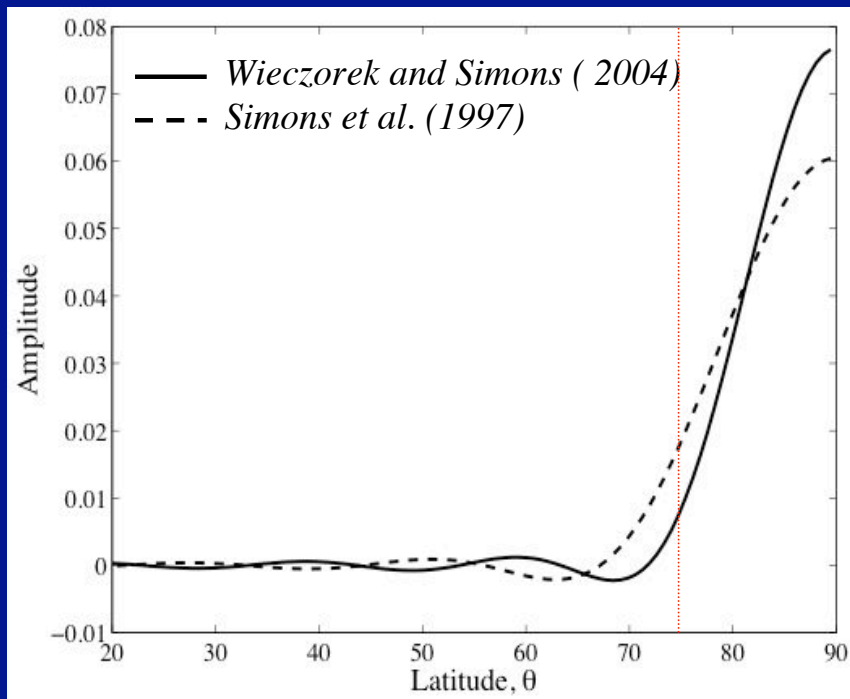


Wieczorek and Simons (2004)

- For a windowed field, the power of a given input spherical harmonic l leaks into the degrees $l - L_{win}$ to $l + L_{win}$
- Because of this, each angular degree l of the localized field depends on degrees between $l - L_{win}$ and $l + L_{win}$
- When working with spectrally truncated data sets, only the degrees $L_{win} \leq l \leq L_{obs} - L_{win}$ are reliable.
- In practice, we ignore the first $L_{win} + 6$ degrees, as these are primarily a result of the global Tharsis signature.

3. Localized admittance calculation

- We use windows that are optimally concentrated within a spherical cap for a given value of L_{win} .



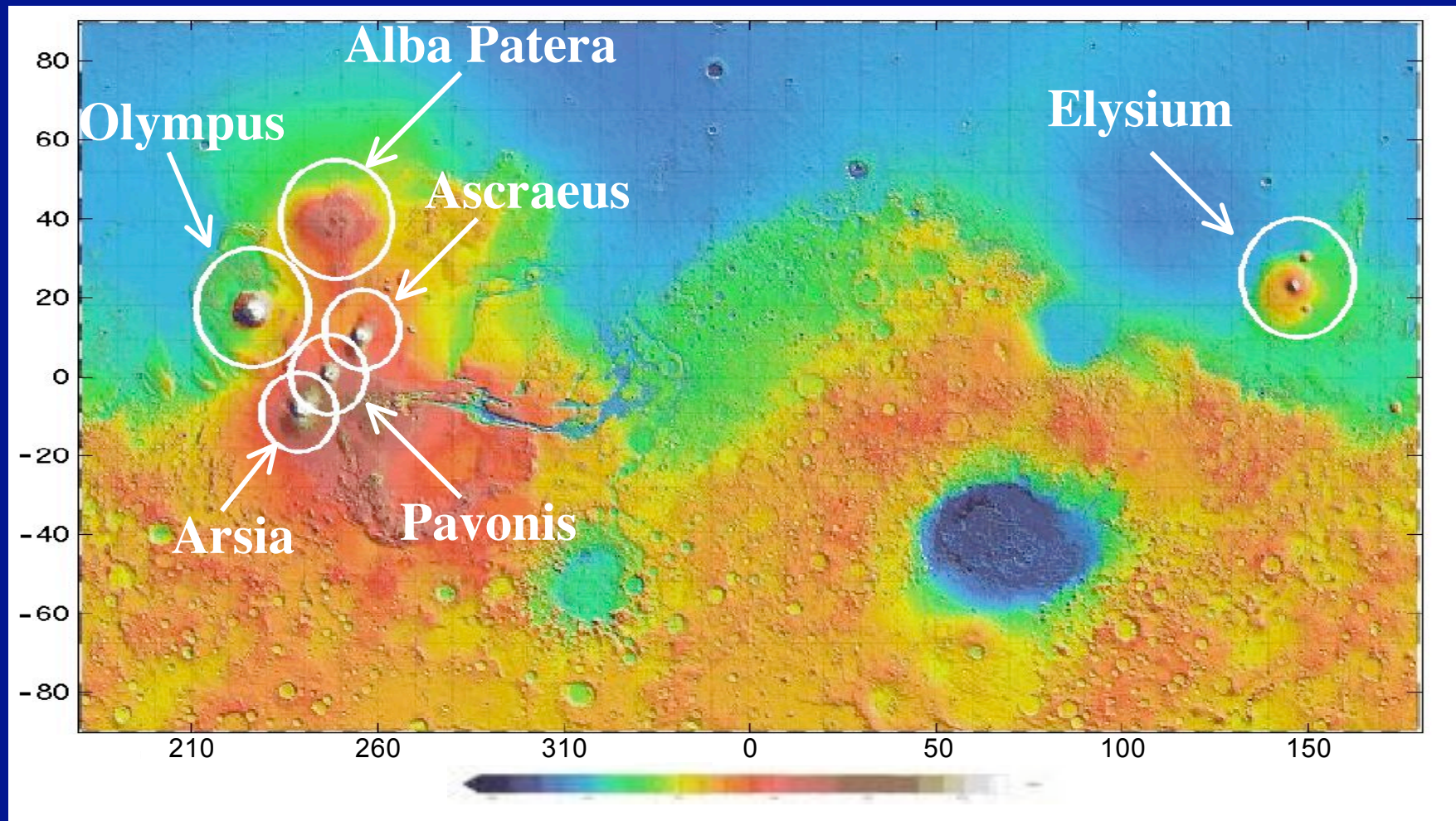
Concentration of energy

- *Wieczorek and Simons, (2004)*

: > 99 %

- *Simons et al. (1997) : 93 %*

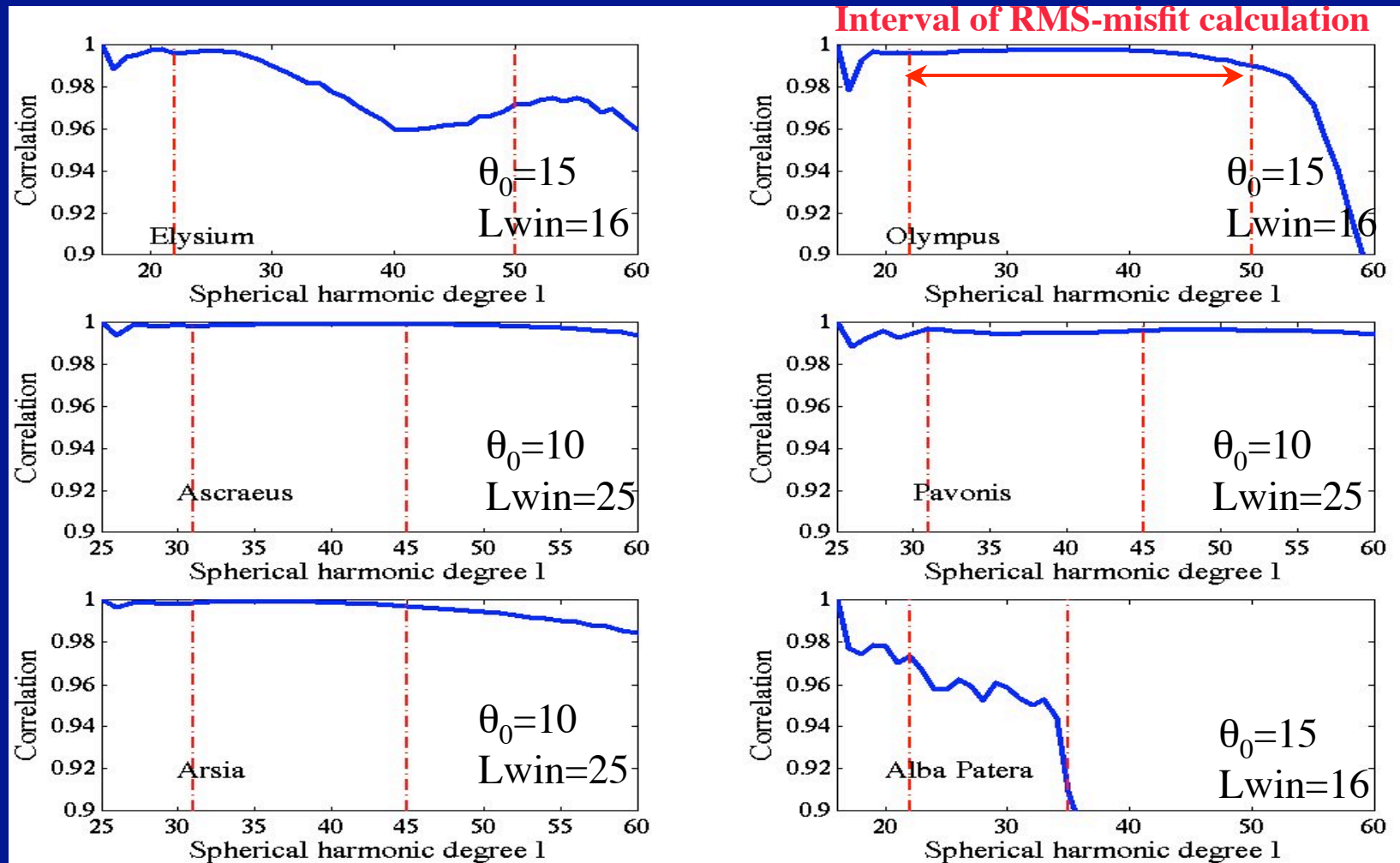
Martian volcanoes localization



(Credit: MOLA Science Team)

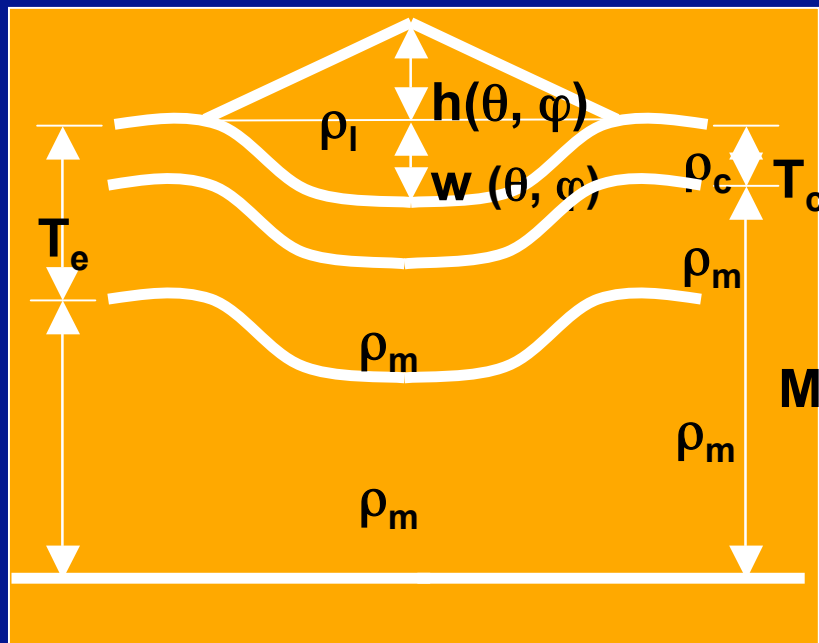
Coherence spectra of the major Martian volcanoes

- Since surface and internal loads are assumed to be correlated, correlations have to be close to 1.



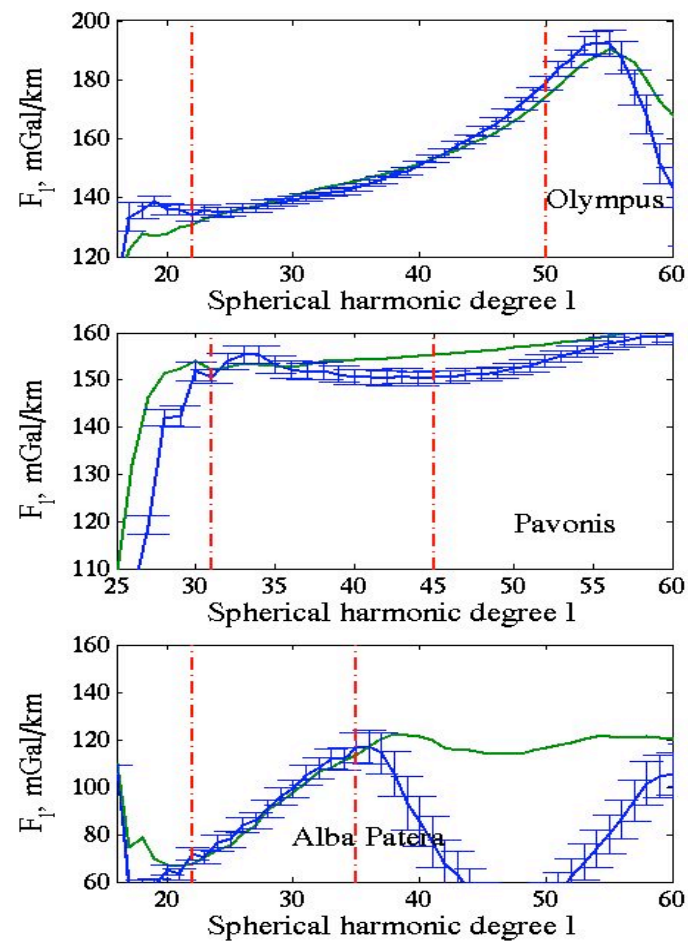
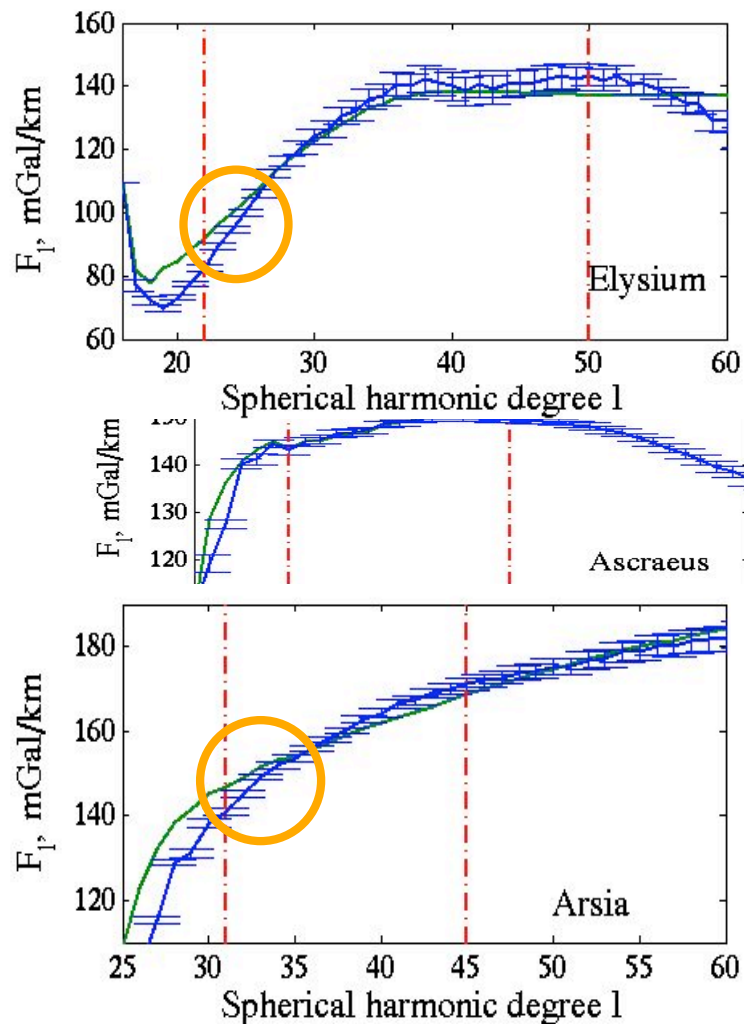
Case I: Modeling surface load only

The surface topography is assumed to be due to volcanically constructed surface loads



ρ_l, ρ_c, T_e, T_c are systematically varied.
 $f=0$.

Admittance spectra of the major Martian volcanoes



How the parameters are constrained?

- Assuming that the error associated to the model is Gaussian, we use marginal probability to show the constraints obtained for one parameter

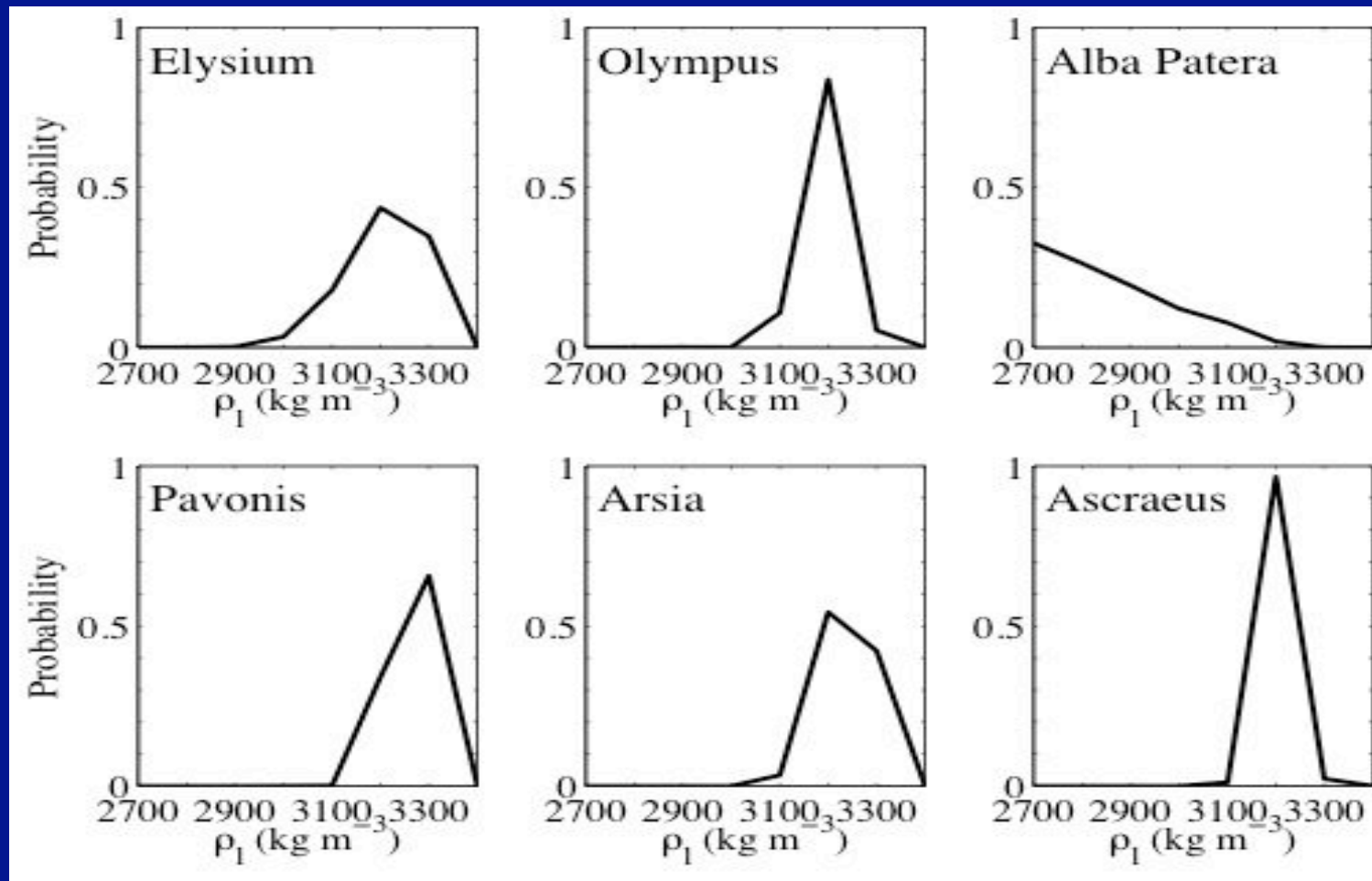
$$P(X_i = x) = C \sum_{j,k,t} \exp\left[-\frac{1}{2} m(X_i, X_j, X_k, X_t)\right]$$

Where m is the misfit function

$$m(X_i, X_j, X_k, X_t) = \frac{1}{\Delta L} \sum_{\ell} \frac{[Z_{\ell}^{obs} - Z_{\ell}^{calc}(X_i, X_j, X_k, X_t)]^2}{\sigma_{\ell}^2}$$

Constraints on the load density

⇒ With the exception of Alba Patera, the load density is constrained to be $\sim 3200 \pm 100 \text{ kg.m}^{-3}$



Densities of the Martian meteorites

With the exception of ALH 84001, the Martian meteorites are believed to come from either Tharsis or Elysium (*McSween 1994,2002*).

EETA 79001B Basaltic Shergottite	3220	LEW 88516 Lherzolite	3400
Los Angeles Basalt	3240	ALH 77005 Lherzolite	3430
QUE 94201 Basalt	3250	Y793605 Lherzolite	3430
Shergotty Basalt	3320	Yamato 000593/749 Clinopyroxenite	3460
Zagami Basalt	3320	Governador Valadares Clinopyroxenite	3460
Dhofar 019 Basalt	3330	Lafayette Clinopyroxenite	3480
EETA 79001A Basaltic Shergottite	3340	Nakhla Clinopyroxenite	3480
Dar al Gani 476/489 Basalt	3360	ALH 84001 Orthopyroxenite	3410
Sayh al Uhaymir 005/094 Basalt	3390	Chassigny Dunit	3580

Neumann et al., 2004

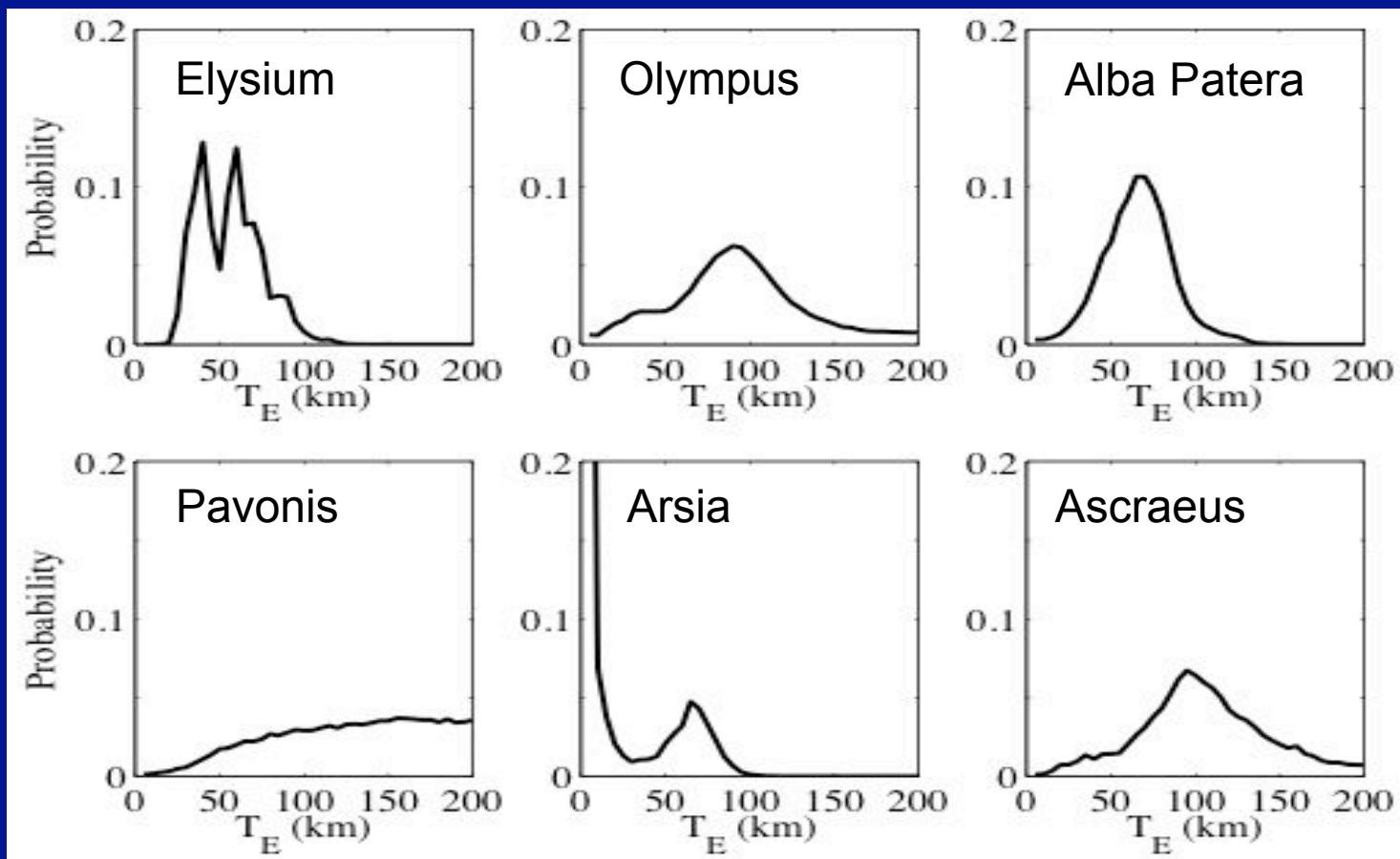
- Pore-free densities are between 3220 and 3580 kg.m⁻³.
- Densities may be reduced by about 150 kg. m⁻³ for a porosity of ~5%.

Constraints on the properties of the Martian volcanoes.

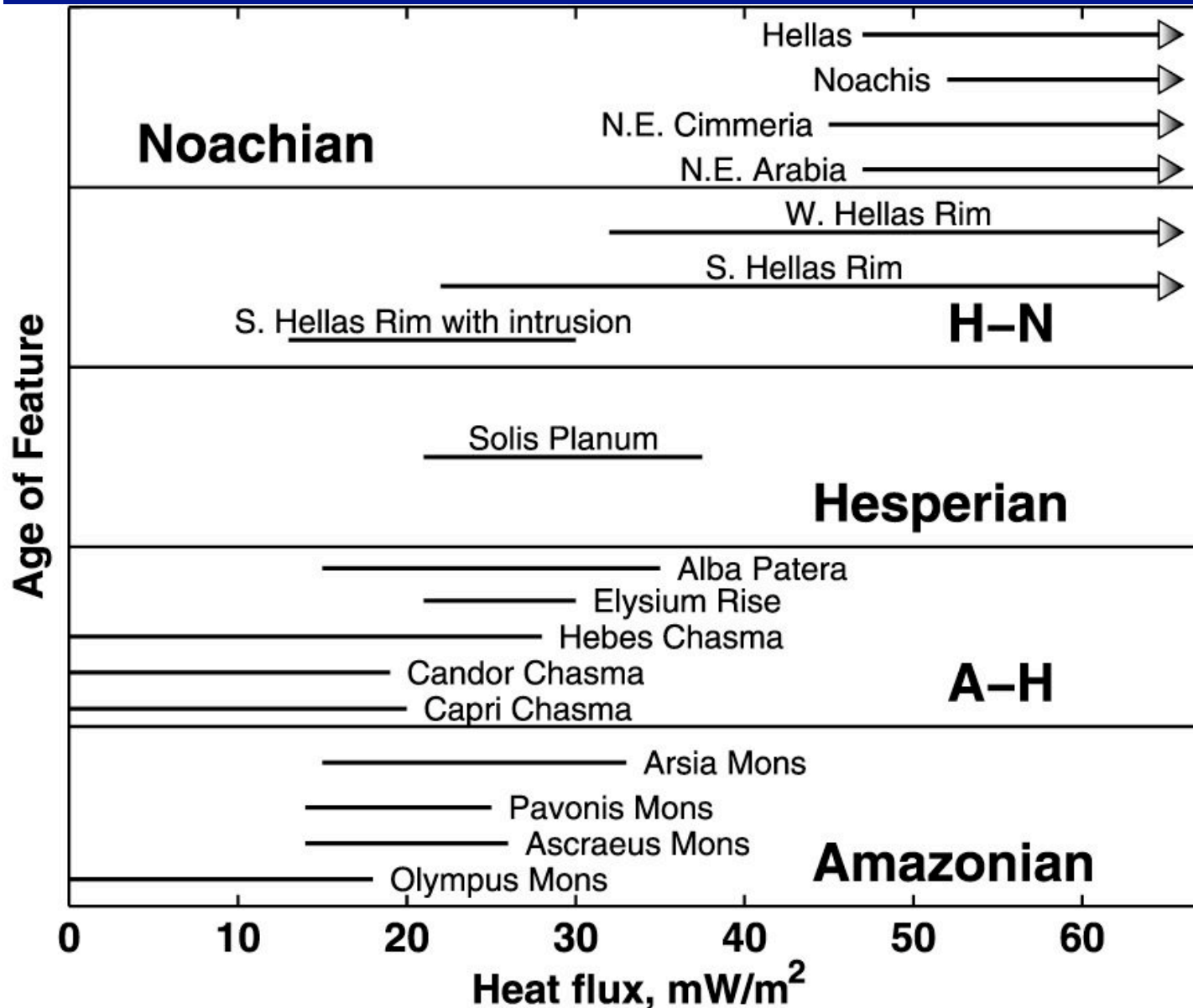
- Higher densities than in previous studies
 - Composition is likely to be similar to the basaltic meteorites
 - Result of a more iron-rich Martian mantle (e.g., Sohl and Spohn, 1997).
- All the volcanoes have a similar density (except Alba Patera)
 - Similar magmatic process over all the planet
 - Alba Patera might be composed of less iron-rich lavas, or alternatively our model is not applicable to this volcano.

Constraints on the elastic thickness

- Elysium: $T_e = 56 \pm 20$ km
- Olympus: $T_e = 93 \pm 40$ km
- Alba Patera: $T_e = 66 \pm 20$ km
- Ascraeus: $T_e = 105 \pm 40$ km



Heat Flux Decreases With Time



Average heat-flux of the Moon:
10 - 24 mW/m^2

Average heat-flux of the Earth:
 $\sim 70 \text{ mW/m}^2$

Current heat-flux of Mars:
 $\sim 15 - 35 \text{ mW/m}^2$

Données gravimétriques et géodésiques: hypothèses et démarches

Masse et volume

Précession des équinoxes
+ J_2

Equation d'Euler (axi-symétrique)

Densité moyenne,
Moments d'inertie

Modèle moyen non unique
(y compris croûte moyenne)

Variation de gravité
Variation de forme

Anomalies
de Bouguer

- Variation d'épaisseur crustale (Pratt)
- Variation de densité crustale (Airy)
- Variation de densité du manteau

Charge

Rhéologie

Estimation régionale
de l'épaisseur de la
lithosphère élastique

Théorie de flexure des
plaques élastiques

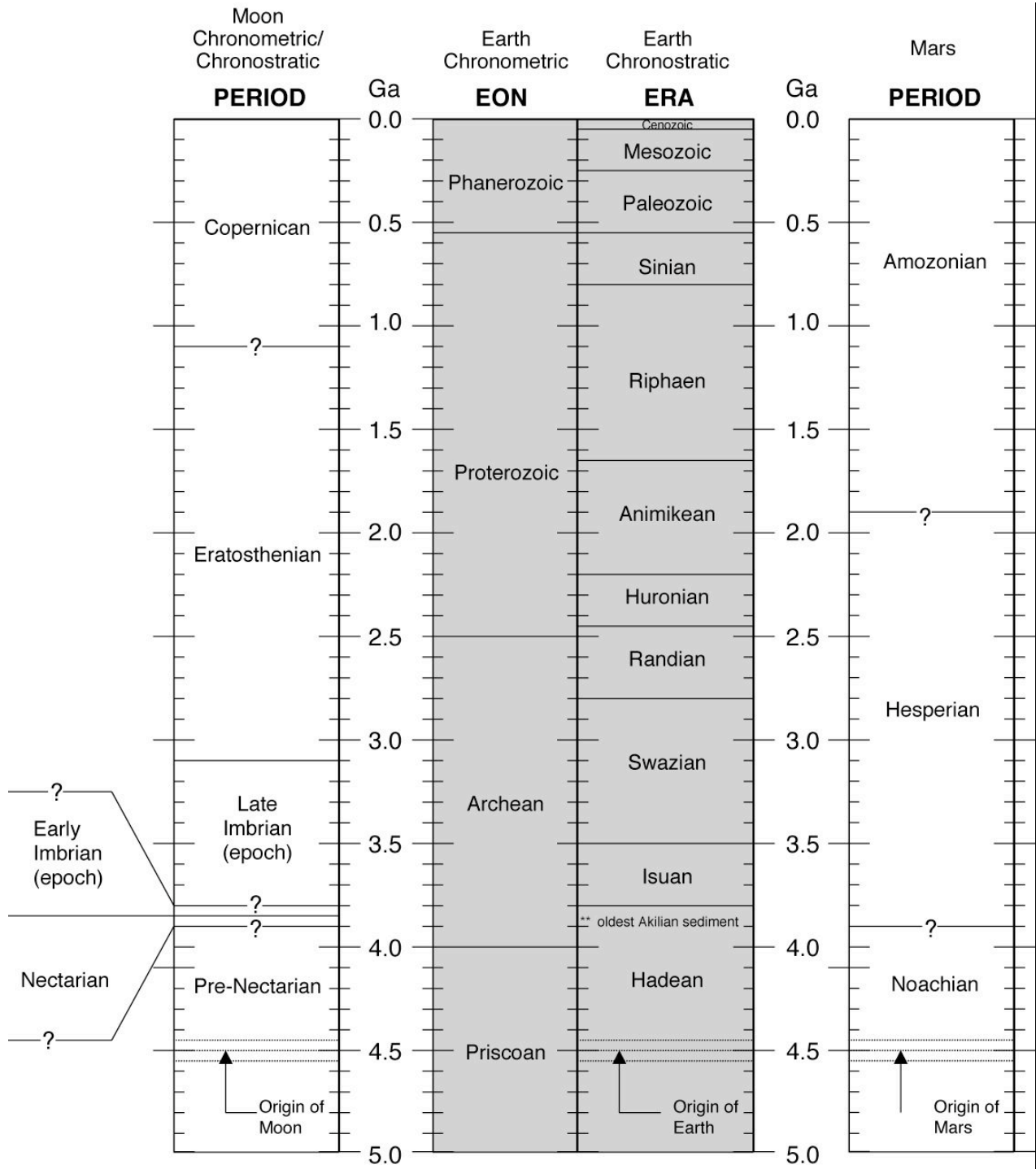
Température
de surface

Profondeur de l'isotherme 650°C

Flux de
chaleur!

What's Next?

- Development of better techniques of analyzing gravity and topography data on a sphere.
 - Spectral estimates and admittances for localized regions on a sphere.
 - Multitaper spectral estimation on a sphere.
- Coupling of thermal evolution models with geophysical constraints
 - Thermal evolution models must give rise to a crust ~40-60 km thick.
 - Thermal evolution models must form the bulk of the Tharsis Rise in the first ~500 Ma of Mars history.
- Origin of topographic dichotomy
 - A result of plate tectonics?
 - Is the density and composition of the northern hemisphere crust the same as that of the southern hemisphere?



Hesperian-Amozonian Boundary
Uncertain by ~1.7 Ga

Noachian-Hesperian Boundary
Uncertain by ~300 Ma