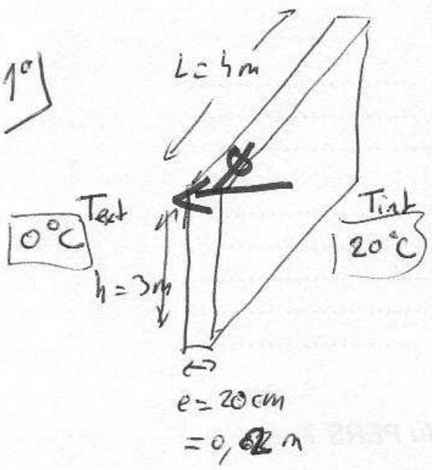
TTT TD3 — A

1°)



$\lambda = 2.10^{-4}$ kcal.m^{-1}s^{-1}K^{-1}

1a) flux de chaleur $\Phi = \lambda S \dfrac{\Delta T}{L}$ en W. ou $\Delta T = \underset{\substack{\uparrow\\ \text{résistance thermique } [°C/W]\ [°K/W]}}{R} \cdot \underset{\hookrightarrow [W]}{\Phi}$

Text < Tint ⇒ le flux est orienté de l'intérieur vers l'extérieur ←

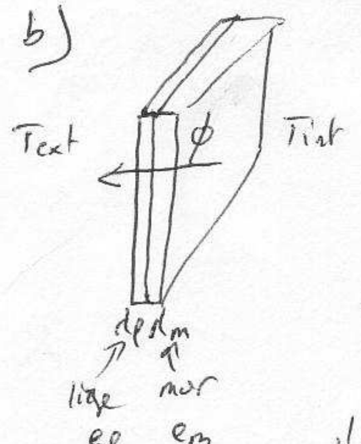
mur simple : $R_m = \dfrac{L}{\lambda S} = \dfrac{e}{\lambda . h . L} = \dfrac{0,2}{2.10^{-4} \times 4,18.10^3 \times 4 \times 3} = \underline{\underline{2.10^{-2}\ °C/W}}$

λ [J/msK ≡ W/mK]

donc $\Phi = \dfrac{\Delta T}{R} = \dfrac{T_{int} - T_{ext}}{R} = \dfrac{20}{2.10^{-2}} = 10^3\,W = \underline{\underline{1\ kW}}$

NB : si T, λ connues ⇒ R → Φ
si Φ, λ connus ⇒ R → T
si T, Φ connus ⇒ R → λ

b)



Hypothèse murs en série : $\Delta T = R\phi$ avec $R = \Sigma R_i$

$R = \dfrac{e_\ell}{\lambda_\ell . h . \ell} + \dfrac{e_m}{\lambda_m . h . \ell} = \dfrac{0,02}{25.10^{-2} \times \dfrac{4,18.10^3}{3600} . 4 \times 3} + \underbrace{2.10^{-2}}_{R_m}$

λ_ℓ [kcal/hmK] = 0,25

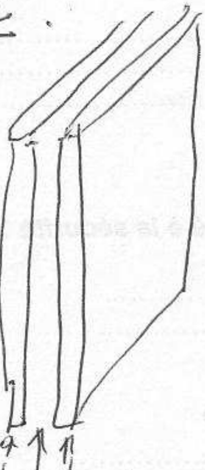
$= 5,7.10^{-3} + 2.10^{-2}$

$R = 25,7.10^{-3}$ °C/W

d'où $\phi = \dfrac{\Delta T}{R} = \dfrac{20}{25,7.10^{-3}} = 778,2\,W = 0,778\,kW$

⇒ réduction de 22% du flux !

1c.



air immobile ⇒ pas de convection !

$$R = \sum R_i = 2\,R_m + R_a$$

$$= \frac{2\,e_m}{\lambda_m . S} + \frac{e_a}{\lambda_a . S}$$

$$= 2 \times \frac{8.10^{-2}}{2.10^{-4} . 4180 \times 4 \times 3} + \frac{4.10^{-2}}{22{,}6.10^{-3} \times \frac{4180}{3600} \times 4 \times 3}$$

λ_m λ_a, e_m.

air $\lambda_a = 22{,}6.10^{-3}$ kcal/hmK $R = \underline{0{,}143}$ C/W.

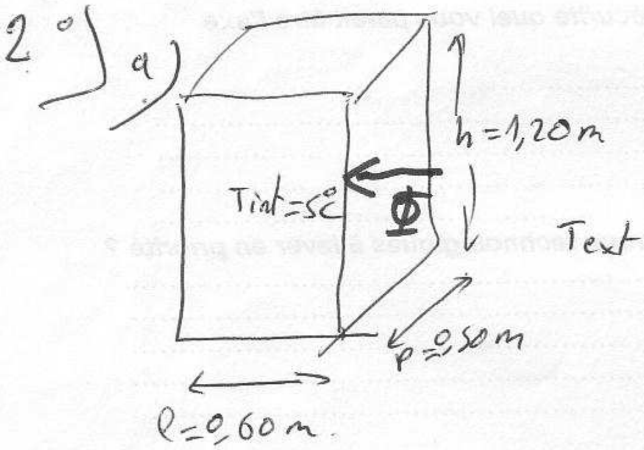
$e_a = 4cm = 4.10^{-2}$ m.

$$\Phi = \frac{\Delta T}{R} = \frac{20}{0{,}143} = \underline{139{,}8\ W}.$$

⇒ diminution de 86 % !!! (pour la même épaisseur !!)

2°) a)

1 paroi = 3 mm de plastique $\lambda = 3{,}1.10^{-2}$ cal/msK



⇒ hypothèse du mur simple sur chaque face.

Text = 20°C.

$$R = \frac{e}{\lambda . S} \Rightarrow \text{il faut calculer S.}$$

si on néglige l'effet d'épaisseur : $S = 2 \times hl + 2 \times hp + 2 \times lp$

$S = 2 \times 1{,}2 \times 0{,}6 + 2 \times 1{,}2 \times 0{,}5 + 2 \times 0{,}6 \times 0{,}5$

$S = 3{,}24\ m^2$

$R_1 = \frac{e}{\lambda S_1}$, $R_2 = \frac{e}{\lambda S_2}$ ...

$\frac{1}{R_1} = \frac{\lambda S_1}{e}$; $\frac{1}{R_2} = \frac{\lambda S_2}{e}$ ...

$\frac{1}{R_T} = \sum \frac{1}{R} = \frac{\lambda}{e} \sum S$.

$\boxed{R_T = \frac{e}{\lambda \sum S}}$

$$\text{et } R = \frac{3.10^{-3}}{3{,}1.10^{-2} \times 4{,}18 \times 3{,}24} = \underline{7{,}15.10^{-3}\ C/W}.$$

$$\text{et } \Phi = \frac{\Delta T}{R} = \frac{20-5}{7{,}15.10^{-3}} = \underline{2097\ W}$$

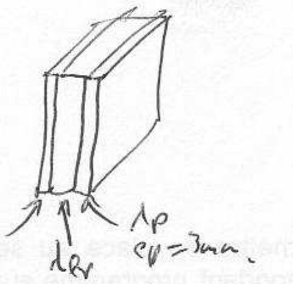
le groupe doit évacuer cette chaleur, en supposant le rendement de 1

⇒ on met à plat le réfrigérateur !!!

$P = \underline{2097\ W}$

C

2b 1 paroi =



$e_p = 3mm$ $e_{lv} = 4cm$.

* pas d'effet d'épaisseur ⇒ S inchangé = 3,24 m²

$$R = 2R_p + R_{lv} = 2 \times 7{,}15 \times 10^{-3} + \frac{4.10^{-2}}{1.10^{-2} \times 4{,}18 \times 3{,}24}$$

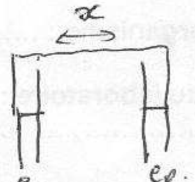
$$R = 14{,}3.10^{-3} + 295{,}4.10^{-3}$$

$$R = 0{,}3097 \ °C/W.$$

$$P = \Phi = \frac{\Delta T}{R} = \frac{15}{0{,}3097} = \underline{48{,}4.\ W}$$

* effet d'épaisseur ⇒ recalculer S.

$e_p = 4{,}6$ cm, soit 9,2 cm au total de réduction des dimensions.



donc $l = 0{,}60 - 9{,}2.10^{-2} = 0{,}508 m$

$h = (120 - 9{,}2).10^{-2} = 1{,}108 m$.

$p = (50 - 9{,}2)\ 10^{-2} = 0{,}408 m$ ⇒ $S = 2lh + 2lp + 2hp = 2{,}44 m^2$.

Pour R, seule la surface à changer $S_1 \rightarrow S_2$.

$$R_{T1} = \frac{e}{\lambda . S_1} \qquad R_{T2} = \frac{e}{\lambda S_2} = R_{T1} \frac{S_1}{S_2}$$

$$R_{T2} = 0{,}3097 \times \frac{3{,}24}{2{,}44} = 0{,}411.\ °C/W$$

$$P = \phi = \frac{\Delta T}{R_{T2}} = \underline{\underline{36{,}5\ W}}$$ ⇒ 25% d'erreur !!!

* effet d'épaisseur

on pourrait aussi la moyenne $S = \frac{S_1 + S_2}{2} = \frac{3{,}24 + 2{,}44}{2} = 2{,}84 m^2$

$$R_{T3} = 0{,}353\ °C/W$$

$$P = \phi = 42{,}4\ W$$ ⇒ 12% d'erreur !!

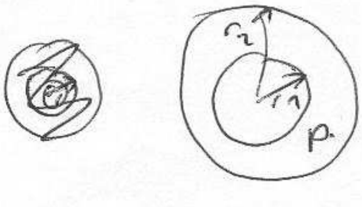
⇒ notion de facteur de forme en abaque.

$$\phi = \lambda F (T_1 - T_2) . [W/m^2] \qquad \equiv F \propto \frac{S}{L}$$

↳ facteur de forme en m

3)

10



a) $\nabla(\lambda \vec{\nabla} T) + p - \rho c \frac{\partial T}{\partial t} = 0$

stationnaire $\Rightarrow \frac{\partial T}{\partial t} = 0$

λ constante $\Rightarrow \lambda \nabla(\vec{\nabla} T) + p = 0 \Rightarrow \nabla(\vec{\nabla} T) + \frac{p}{\lambda} = 0.$

$\Delta T + \frac{p}{\lambda} = 0.$

coord. cylindrique : $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial \varphi} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

symétrie $T = T(r) \rightarrow \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{p}{\lambda} = 0$

on multiplie par $r \Rightarrow \underline{r\frac{d^2 T}{dr^2} + \frac{dT}{dr} = -\frac{p}{\lambda} r}$

b). équation de la forme $uv' + u'v = g(r).$ $u = r, v = \frac{dT}{dr}.$

$\Rightarrow \int uv' + u'v = \int g(r) \Leftrightarrow [uv] = \int g(r).$

$\Leftrightarrow r \cdot \frac{dT}{dr} = -\frac{p}{2\lambda} \cdot r^2 + A$

donc $\frac{dT}{dr} = -\frac{p}{2\lambda} \cdot r + \frac{A}{r}. \Rightarrow \boxed{T(r) = -\frac{pr^2}{4\lambda} + A\ln r + B.}$

c) barre = cylindre avec $r_1 \rightarrow 0$.

or $\lim_{r \to 0} T(r) = \infty \Rightarrow$ impossible $\Rightarrow A = \emptyset.$

barre : $T(r) = -\frac{pr^2}{4\lambda} + B.$

CL: $T(r_2) = T_2 \Rightarrow B = T_2 + \frac{p r_2^2}{4\lambda}.$

donc $T(r) = -\frac{pr^2}{4\lambda} + T_2 + \frac{p r_2^2}{4\lambda} = \underline{T_2 + \frac{p}{4\lambda}(r_2^2 - r^2)}$

loi de Fourier $\vec{\phi} = -\lambda \overrightarrow{\nabla T}$

(E

or en cylindr. $\vec{\nabla} f = \frac{\partial f}{\partial r} \cdot \vec{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta} \vec{u_\theta} + \frac{\partial f}{\partial z} \cdot \vec{u_z}$

$\left(\text{et } \nabla \vec{F} = \frac{1}{r}\frac{\partial (r A_r)}{\partial r} + \frac{1}{r} \cdot \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \right)$

$\Rightarrow$ donc $\phi = -\lambda \frac{\partial T}{\partial r} = +\lambda \cdot \frac{p \cdot r}{2\lambda} = \underline{\underline{\frac{p \cdot r}{2}}}$ $[W \cdot m^{-2}]$.

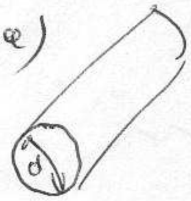
d) $T(r) = T_2 + \frac{p}{4\lambda}(r_2^2 - r^2) \Rightarrow T_0 = T(r=0) = \underline{\underline{T_2 + \frac{p}{4\lambda} \cdot r_2^2}}$

donc $\underline{\Delta T = T_0 - T_2 = \frac{p}{4\lambda} r_2^2}$ $\quad > 0$ si $p > 0$ (source) ; < 0 si $p < 0$ (puit)

6) flux est $\Phi = \phi \cdot S = \phi \cdot 2\pi r_2 L = \frac{p \cdot r_2}{2} \times 2\pi r_2 L = p \cdot \pi r_2^2 \cdot \underset{\substack{\uparrow \\ \text{longueur de la barre!}}}{L}$

$= p \cdot$ Volume

4) a)



$P_{\text{elec}} = U \cdot I = \underset{\substack{\uparrow \\ \text{résistance électrique}}}{R} \cdot I^2 = \rho \frac{L}{\underset{\substack{\parallel \\ \frac{\pi d^2}{4}}}{S}} \cdot I^2 = \frac{4\rho L}{\pi d^2} \cdot I^2$

le volume de la barre est $V = \frac{\pi d^2}{4} \times L \Rightarrow p = \frac{P}{V} = \frac{4\rho L \cdot I^2}{\pi d^2} \times \frac{4}{\pi d^2 L}$

$\underline{p = \frac{P}{V} = \frac{16 \rho \cdot I^2}{\pi^2 d^4}}$

cuivre: $p_c = \frac{16 \times \overset{\mu\Omega \to \Omega \cdot m}{\overset{\parallel}{1{,}7 \times 10^{-6}}} \cdot \overset{\parallel}{10^{-2}} \times I^2}{\pi^2 \quad d^4} = 2{,}76 \cdot 10^{-8} \frac{I^2}{d^4}$

$\Rightarrow$ environ 3500

graphite: $p_g = 16 \times \frac{6 \cdot 10^{?} \cdot 10^{-6} \cdot 10^{-2} I^2}{\pi^2 \; d^4} = 9{,}73 \cdot 10^{-5} \frac{I^2}{d^4}$

b) cf exercice 3 $\Rightarrow \Delta T = \frac{p}{4\lambda} \cdot r^2 \Rightarrow \Delta T = \frac{p}{4\lambda}\left(\frac{d}{2}\right)^2 = \frac{p\, d^2}{16\lambda}$

$= \frac{16 \rho I^2 d^2}{\pi^2 d^4 \, 16 \lambda}$

$\Delta T = \frac{\rho I^2}{\pi^2 d^2} \cdot \frac{1}{\lambda}$

donc $\Delta T \nearrow$ si $\rho \nearrow$, si $I \nearrow$, si $d \searrow$, si $\lambda \searrow$

cuivre : $\Rightarrow \Delta T = 4{,}12 \cdot 10^{-12} \left(\frac{I}{d}\right)^2$

graphite $\Rightarrow \Delta T = \cancel{4{,}15 \cdot 10^{-10}} \cancel{\left(\frac{I}{d}\right)^2}\ 3{,}64 \cdot 10^{-7} \left(\frac{I}{d}\right)^2$ un rapport 90 000 !!!

si $I = 500\ A \Rightarrow$ cuivre $\Delta T = 0{,}01\ °C$

$d = 1\,cm \Rightarrow$ graphite $\Delta T = 908{,}98\ °C$.

5°)

−15 °C

$$\frac{\partial^2 T(z,t)}{\partial z^2} - \frac{1}{a}\frac{\partial T(z,t)}{\partial t} = 0.$$

sol (sec) $T_0 = 5°C$

a

Transformée de Laplace

$$\frac{d^2 \Pi(z,p)}{dz^2} - \frac{p}{a}\Pi(z,p) = \frac{T_0}{a}.$$

changement de variable $T^*(z,t) = T(z,t) - T_0$.

$$\Pi^*(z,p) = \Pi(z,p) - \frac{T_0}{p}$$

$$\frac{d^2 \Pi^*(z,p)}{dz^2} - \frac{p}{a}\Pi^*(z,p) = 0.$$

$$\Pi^*(z,p) = Ae^{-hz} + Be^{hz} \qquad h^2 = \frac{p}{a}$$

or $T(z,0) = T_0 \Rightarrow \Pi^*(z,p) = 0$.

$T(0,t) = T_1 \Rightarrow \Pi^*(0,p) = T_1 - T_0/p$

$\Pi^*(z,p) \xrightarrow[z \to \infty]{} 0 \Rightarrow B = 0$.

$z = 0 \Rightarrow \Pi^*(z,p) = \frac{T_1 - T_0}{p} = A\exp(0) = A$

$\Rightarrow \Pi^*(z,p) = \frac{1}{p}\exp \cdot -\sqrt{\frac{p}{a}}\, z$

Laplace

$$\boxed{T(z,t) = T_0 + (T_1 - T_0)\,\mathrm{erfc}\left(\frac{1}{\sqrt{4at}}\, z\right)}$$

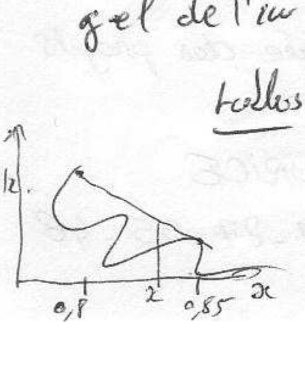
gel de l'eau $\Rightarrow T(x,t) = 0 \Rightarrow \mathrm{erfc}(u) = \frac{-T_0}{T_1 - T_0} = 0{,}25$

tables : $\mathrm{erfc}(0{,}8) = 0{,}257899$. $\Rightarrow \cancel{x = 0{,}8 + \frac{\mathrm{erfc}(0{,}8) - \mathrm{erfc}(0{,}85)}{0{,}8 - 0{,}85} \times \mathrm{erfc}(x)}$

$\mathrm{erfc}(0{,}85) = 0{,}229332$

$\mathrm{erfc}(x) = 0{,}25$ $=$

$\cancel{\mathrm{erfc}(x) = ax + b} \Rightarrow x = \frac{\mathrm{erfc}(x) - b}{a}$.



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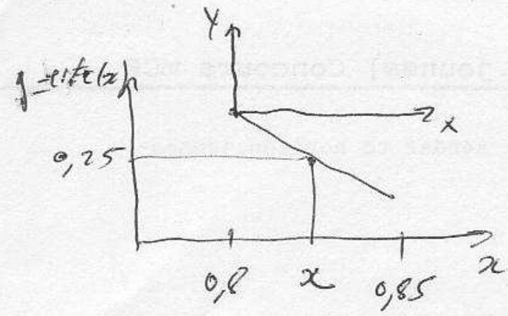


$Y = y - \mathrm{erfc}(0,8)$,

$X = x - 0,8$.

$Y = aX \Rightarrow X = \frac{Y}{a} = \frac{\mathrm{erfc}(x) - \mathrm{erfc}(0,8)}{\frac{\mathrm{erfc}(0,8) - \mathrm{erfc}(0,85)}{0,8 - 0,85}}$

or $\mathrm{erfc}(x+b) = \cancel{\mathrm{erfc}(x)} \cancel{\int_0^{x+b}} = \int_0^x + \int_x^{x+b}$

donc $x = 0,8 + \frac{(\mathrm{erfc}(x) - \mathrm{erfc}(0,8))(0,8 - 0,85)}{\mathrm{erfc}(0,8) - \mathrm{erfc}(0,85)}$ $\cancel{\mathrm{erfc}(x)} +$

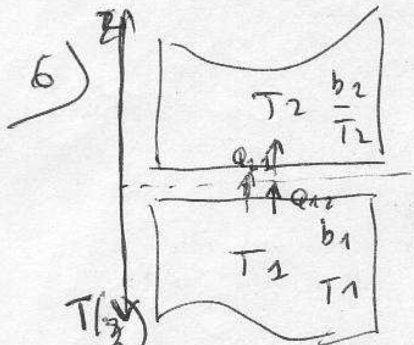
$x = 0,8 + \frac{(0,25 - 0,257899)(0,8 - 0,85)}{0,257899 - 0,224332} = 0,81\cancel{4}$

et $u = \frac{1}{\sqrt{4at}} z \Rightarrow z = u\sqrt{4at} = 0,814 \times 2 \times \sqrt{2,7 \times 10^{-7} \times 15 \times 24 \times 3600}$

$\underline{z = 0,96\ m.}$ (96,30 cm).

NB : sans interpolation $z = \cancel{0,84\ m}$ 94,65 cm $\Rightarrow < 2\%$.

6)



semi-infini $\Rightarrow T(x,t) = T_0 + (T_1 - T_0)\,\mathrm{erfc}\left(\frac{1}{\sqrt{4at}} z\right)$

$\Phi = -\lambda S \left.\frac{dT}{dz}\right|_{z=0} = \frac{\sqrt{\lambda \rho C}\cdot S\,(T_1 - T_0)}{\sqrt{\pi t}} = \frac{b \cdot S (T_1 - T_0)}{\sqrt{\pi t}}$

si $T_1 > T_2 \Rightarrow \Phi$ du milieu 1 vers le milieu 2

$\phi_{1\to 2} = \frac{b_1 \cdot S \cdot (T - T_1)}{\sqrt{\pi t}}$. $\cancel{<}\ 0$.

à la mise en contact :

T moyen. contact.

$\phi_{2\to 1} = b_2 \frac{S \cdot (T - T_2)}{\sqrt{\pi t}}$ $\cancel{>}$ ⊕

conservation des flux : $\phi_{1\to 2} + \phi_{2\to 1} = 0$.

$b_1 \cancel{S}(T - T_1) + b_2 \cancel{S}(T - T_2) = 0$

$$\boxed{T = \frac{b_1 T_1 + b_2 T_2}{b_1 + b_2}}$$

$b_{\text{salle}} \simeq b_{\text{carapace}} \Rightarrow b_1 = b_2 = b$.

$T = \frac{b T_1 + b T_2}{2b} = \frac{T_1 + T_2}{2} \Rightarrow \frac{37,5° + 55}{2} = \underline{46,25\ °C}$.

$b_{\text{métal}} \gg b_{\text{carapace}} \Rightarrow T = \frac{b_2\left(\frac{b_1}{b_2} T_1 + T_2\right)}{b_2\left(\frac{b_1}{b_2} + 1\right)} \rightsquigarrow T \to T_2 = \underline{55\ °C}$