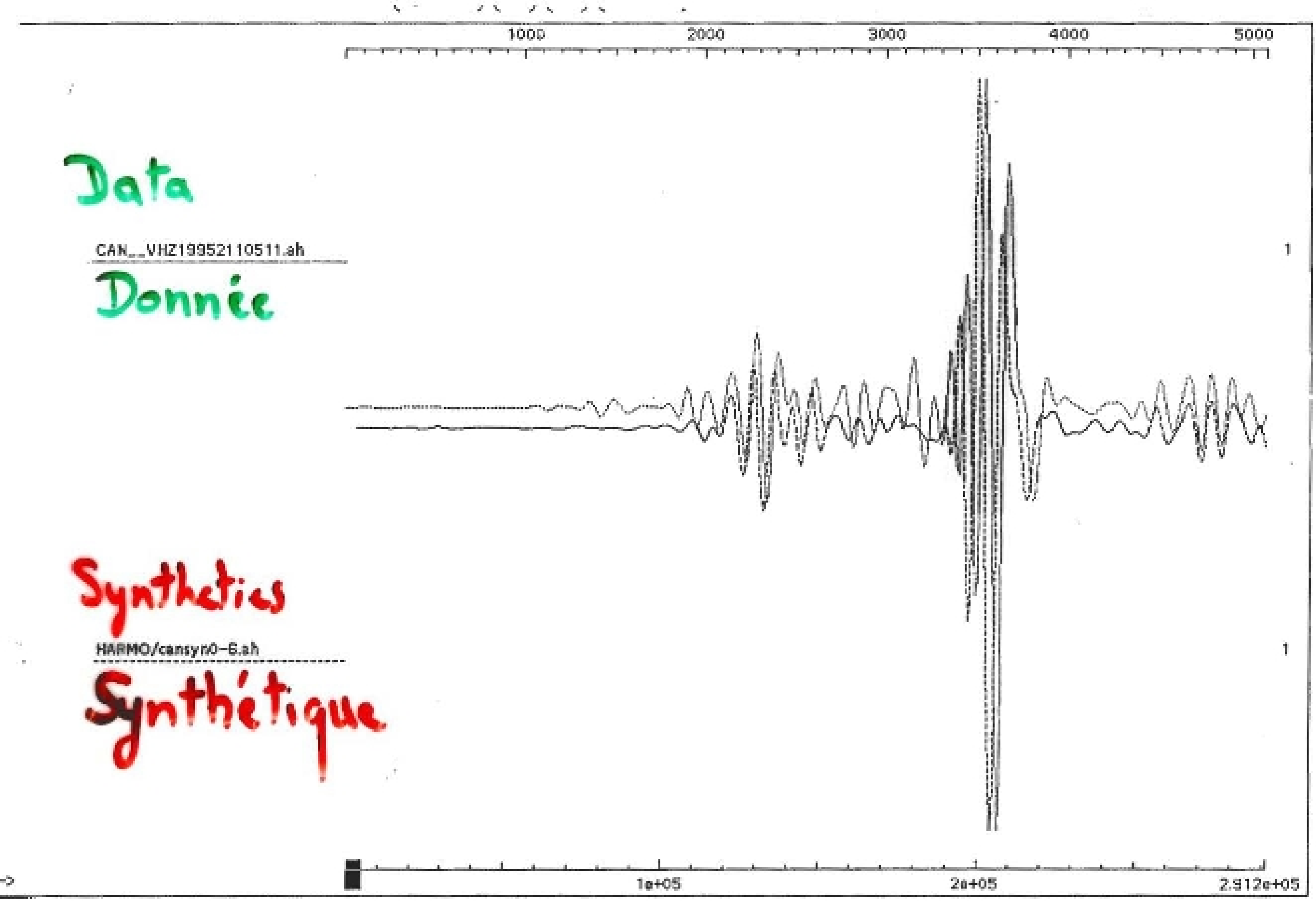
Imageries Globale et Régionale

et Applications Géodynamiques

Jean-Paul Montagner

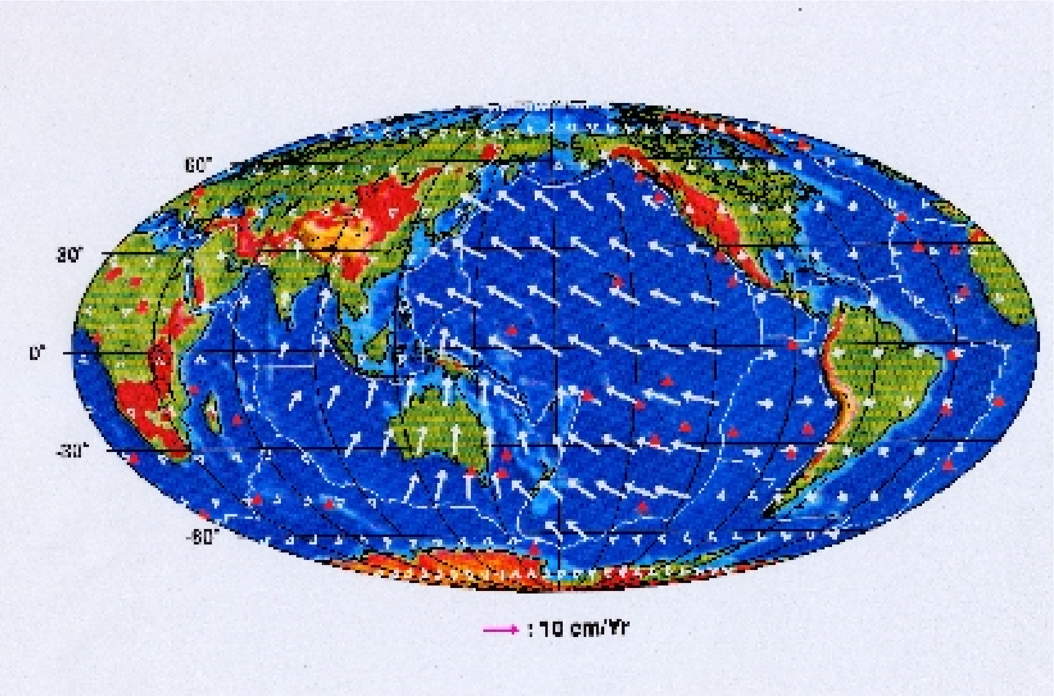
Dept. Sismologie, I.P.G., Paris; France



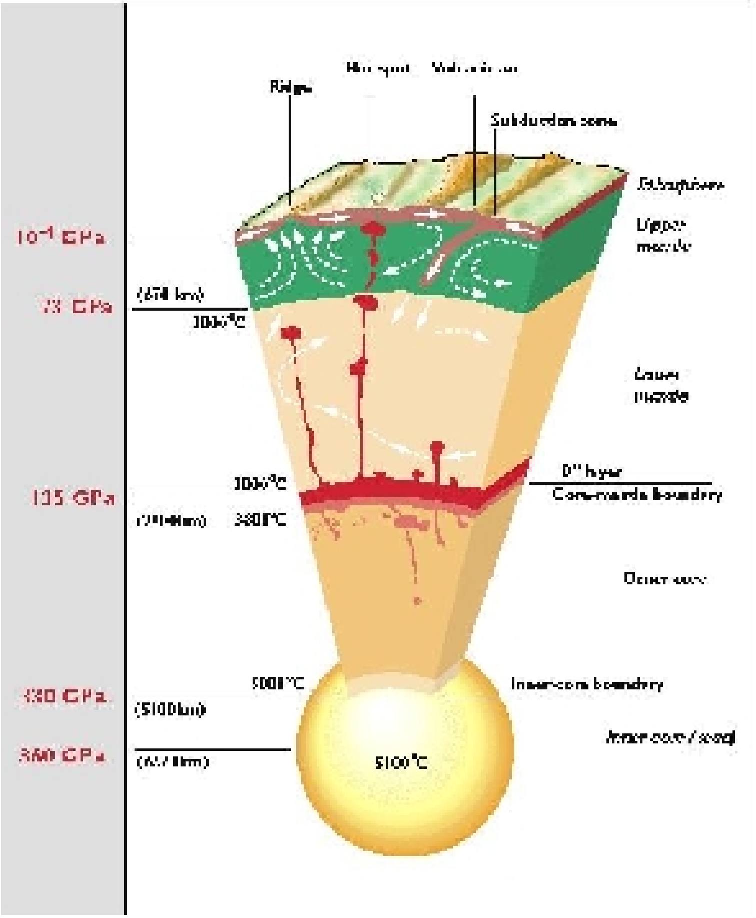
Data
CAN__VHZ19952110511.ah
Donnée
Synthetics
HARMO/cansyn0-6.ah
Synthétique
1000
2000
3000
4000
5000
1
1
1e+05
2e+05
2.912e+05

Structure of the Earth

Plate tectonics



Mantle Convection



Tomographic Technique

- Forward Problem: Theory $\mathbf{d}=\mathbf{g}(\mathbf{p})$

d data space, **p** parameter space

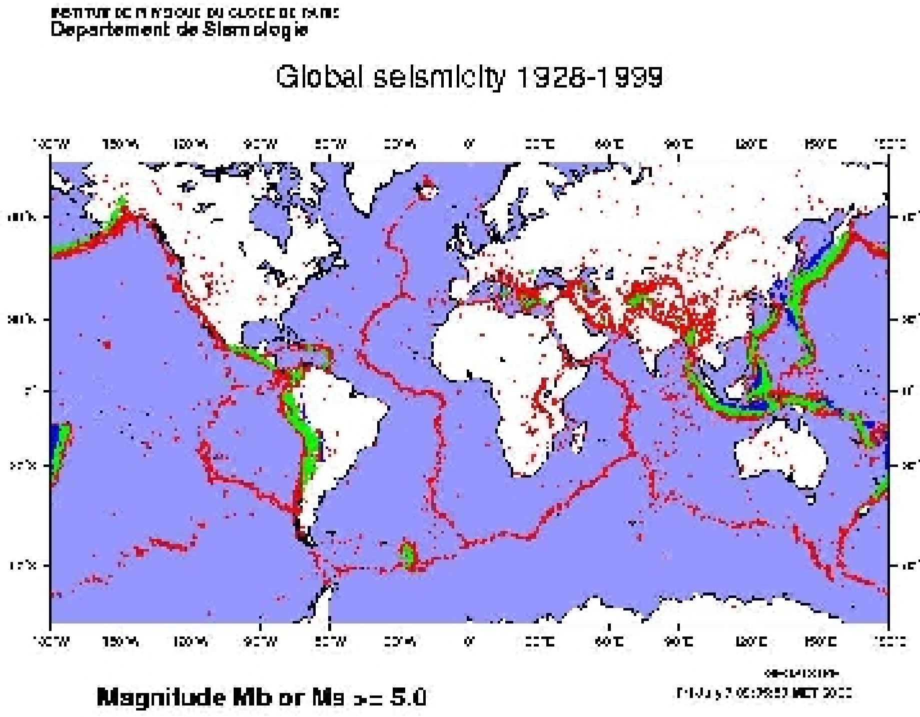
- Reference Earth model p_0:

$d_0 = g(p_0)$

- Kernels $\partial g/\partial p$
- Cd function (or matrix) of covariance of data

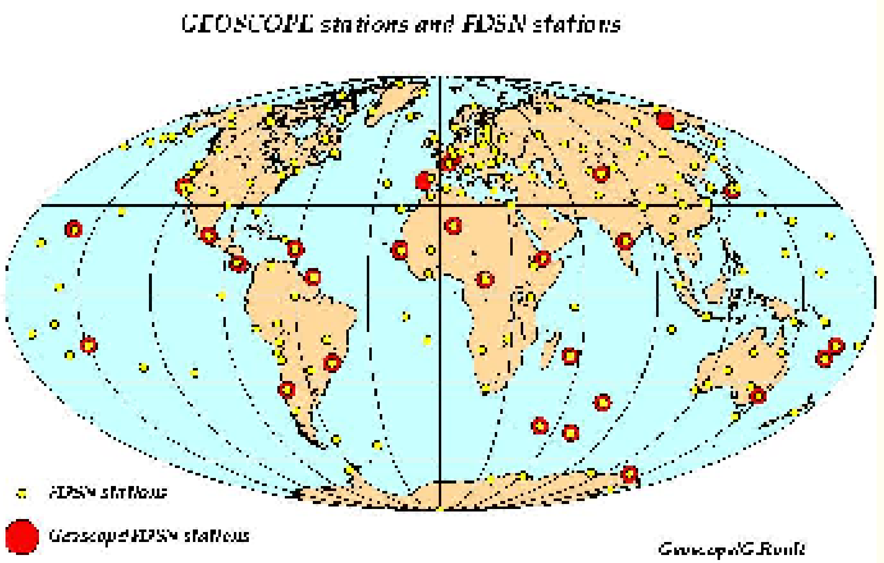
- Inverse Problem: $p-p_0 = \underline{g}^{-1} (d-d_0)$

- C_{p0} a priori Covariance function of parameters
- C_{pf} a posteriori Covariance function of parameters
- R Resolution



Des récepteurs

Des sources



Différents types de données

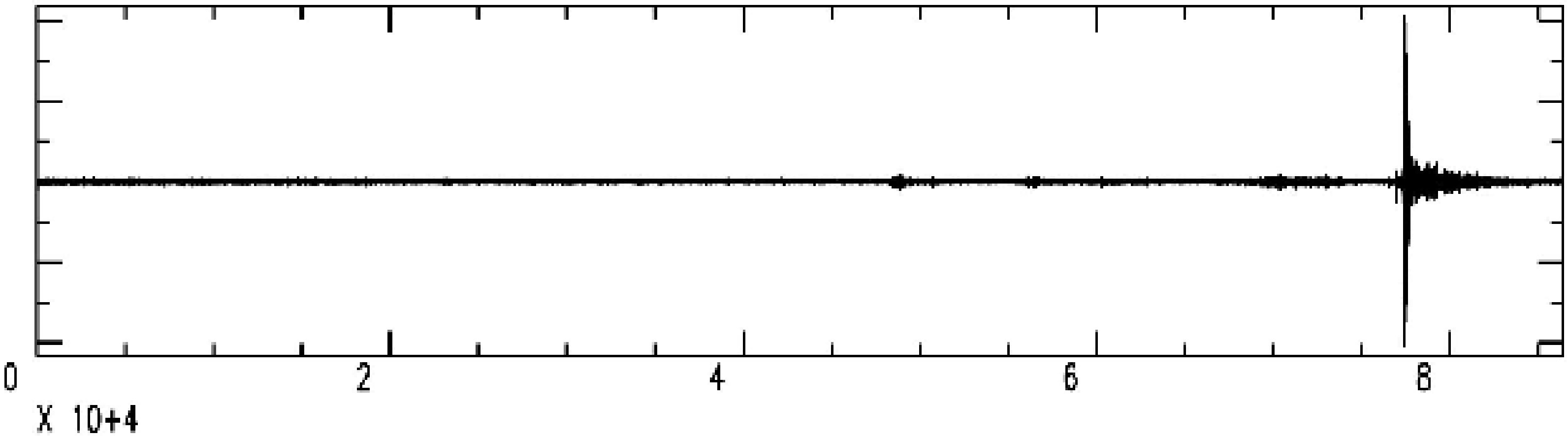
-Ondes de volume (temps d’arrivée) P, S, …..

-Ondes de surface (Rayleigh, Love)

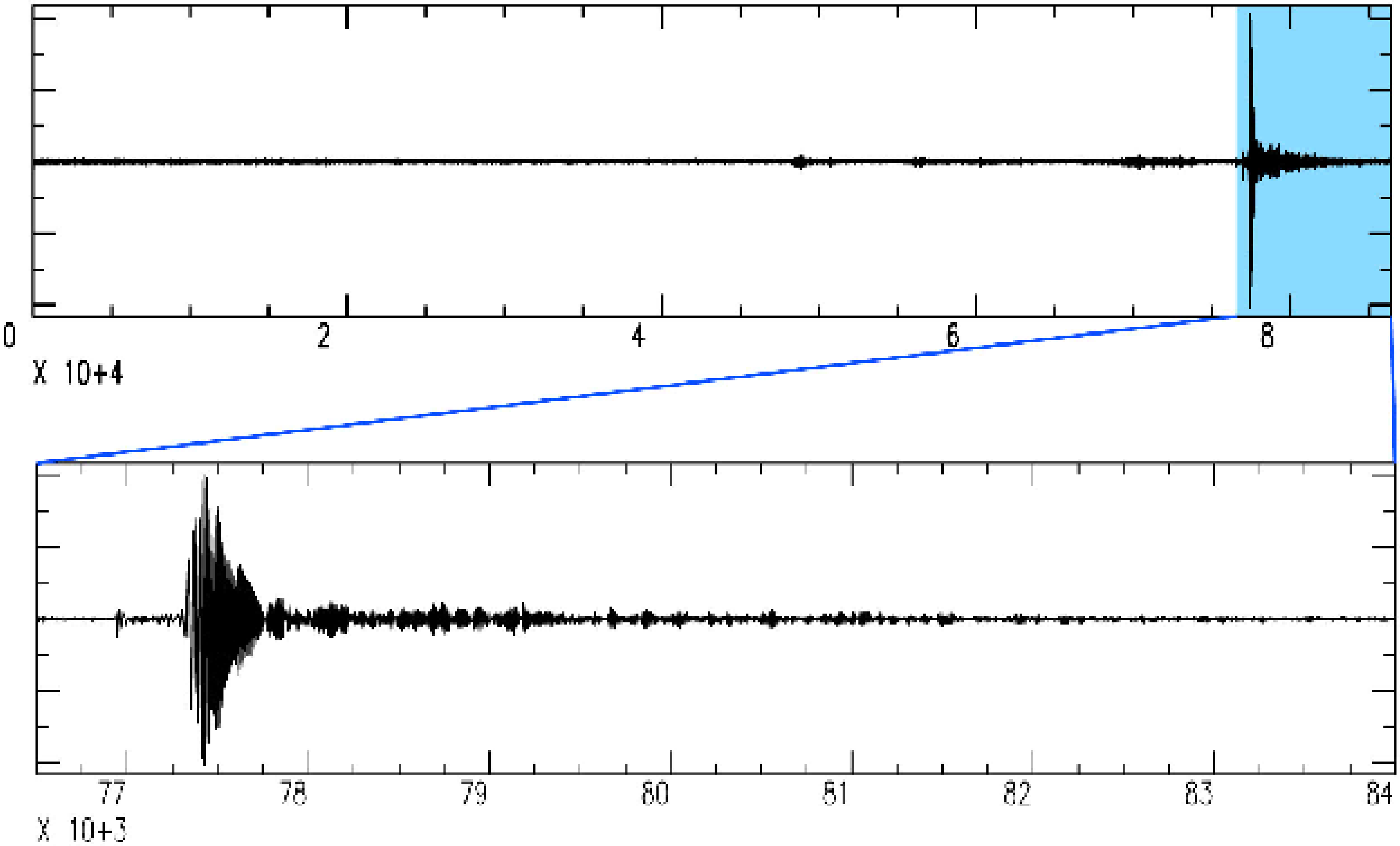
-Modes propres de la terre

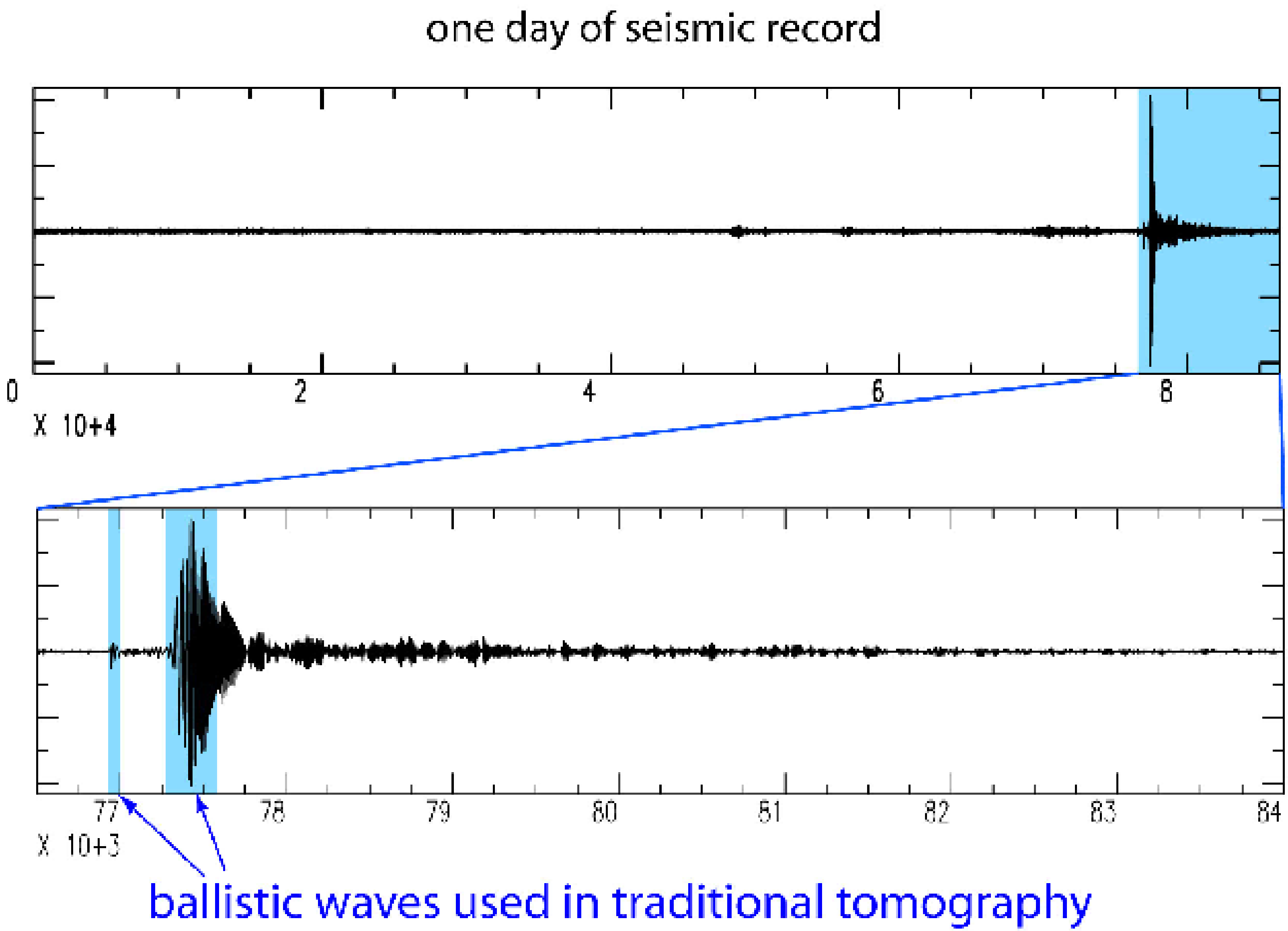
-“Bruit de fond” (1-20s microsismique, >200s “seismic hum”)

one day of seismic record

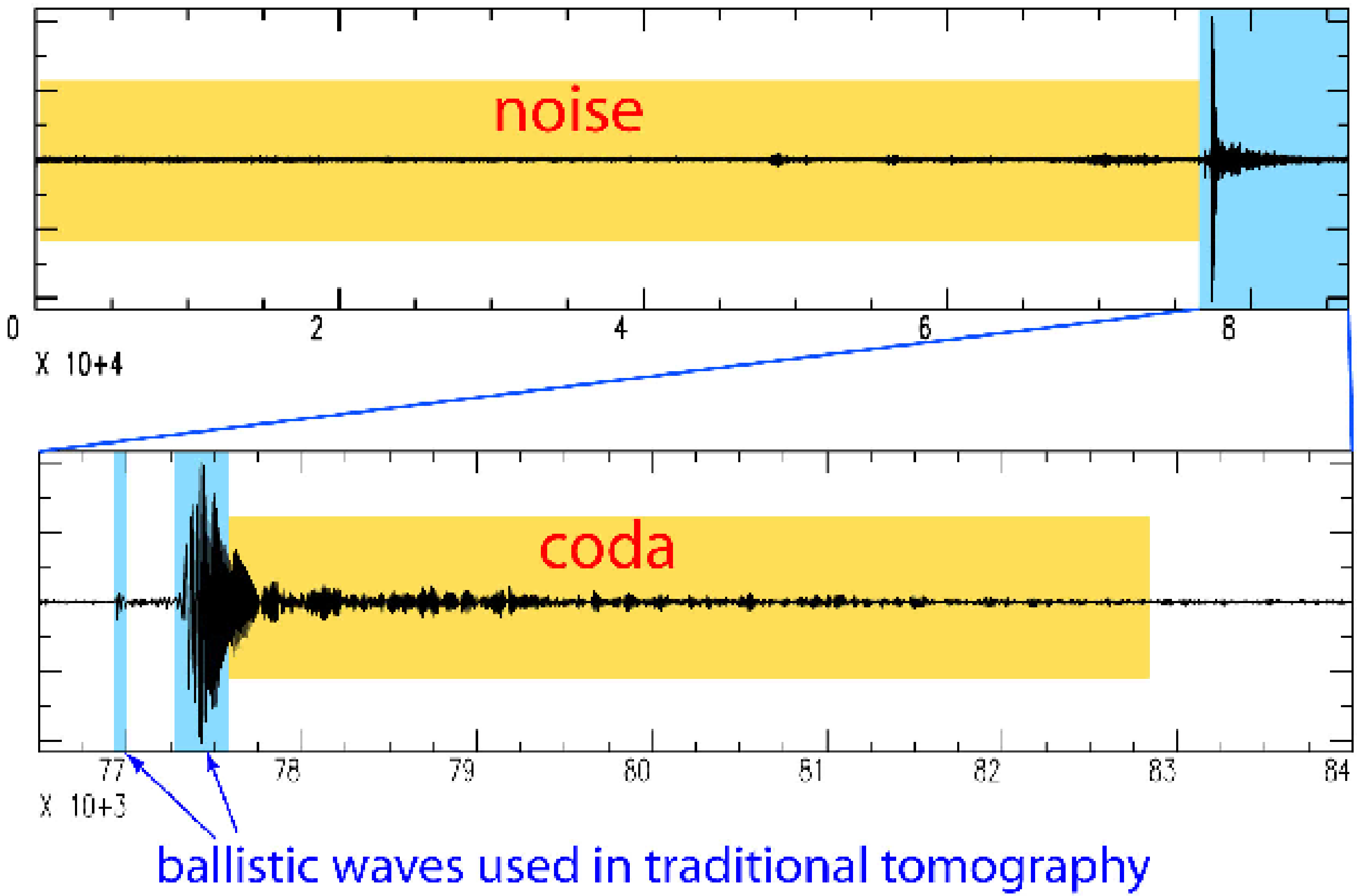


one day of seismic record

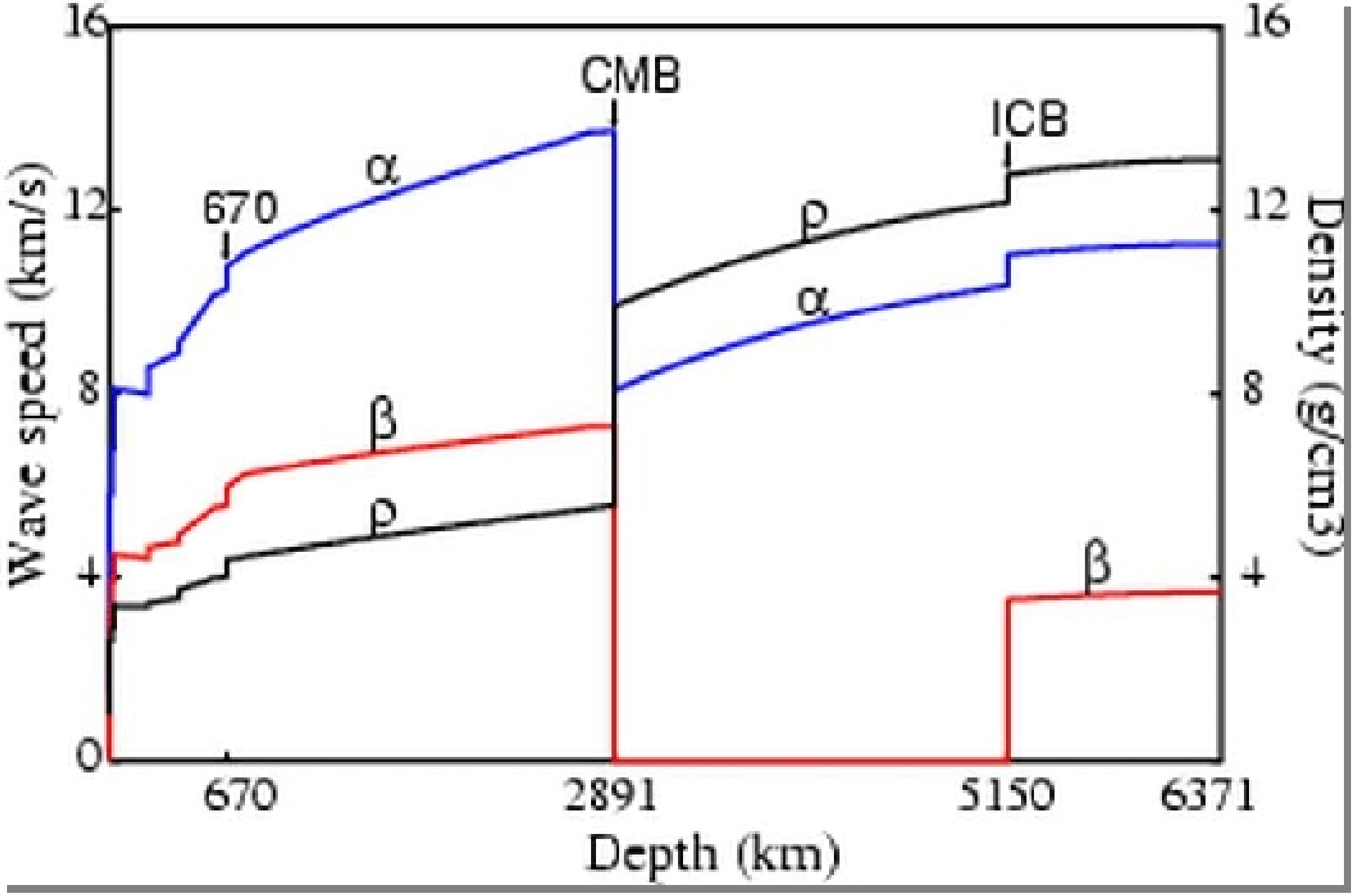



one day of seismic record
0
2
4
6
8
X 10+4
77
78
79
80
51
82
83
84
X 10+3
ballistic waves used in traditional tomography

one day of seismic record



1D- Reference Earth Model: PREM

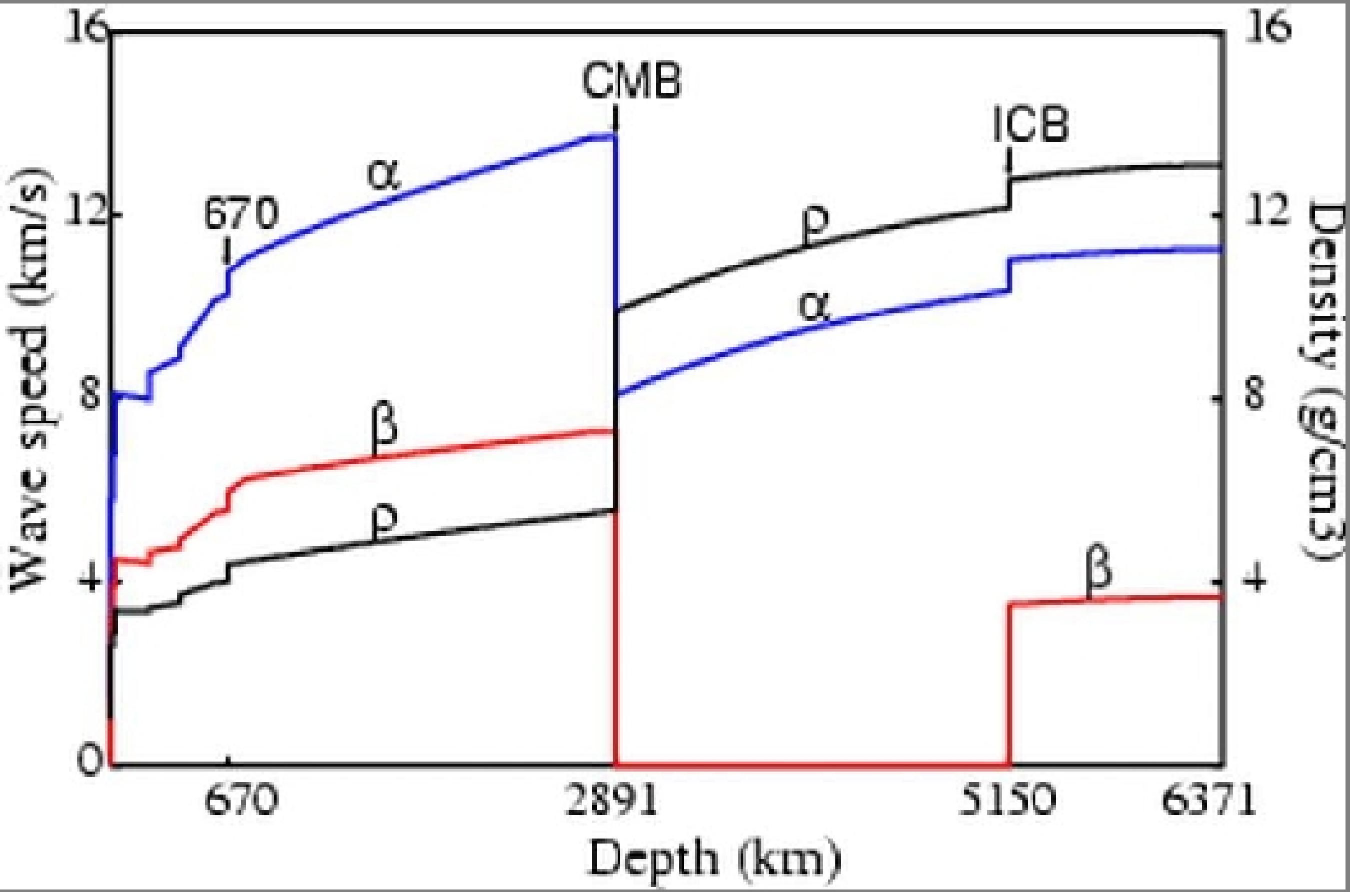


- Normal mode data
- Body wave data (travel times)

∀ ρ, Vp, Vs

∀ ρ, Vph, Vsv, Vpv, Vsh, η

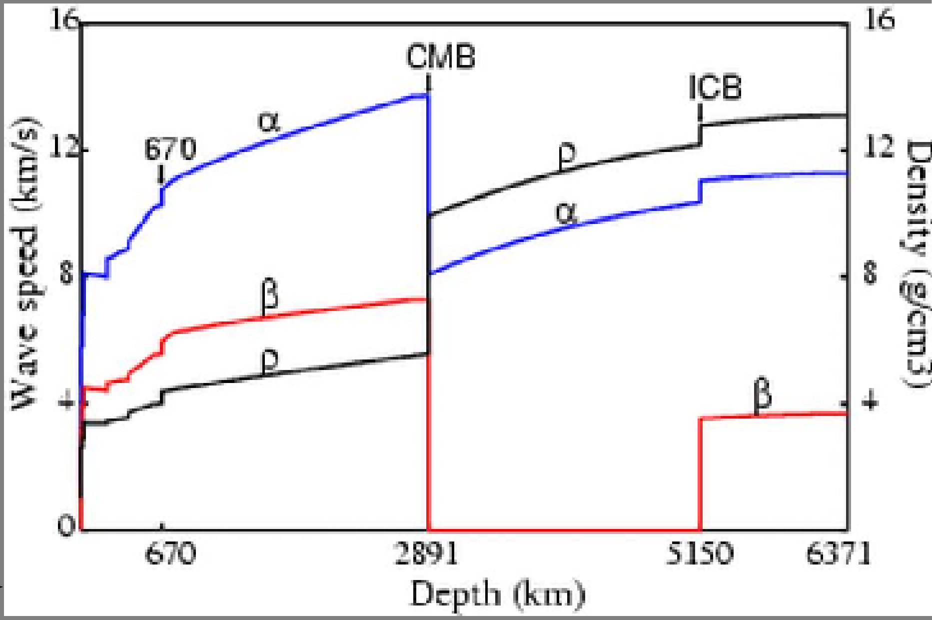
1D- Reference Earth Model: PREM



- Crust
- Lithosphere
- Asthenosphere
- Transition zone
- Lower mantle
- D'' layer
- Outer core
- Inner core

1D- Reference Earth Model

- Normal mode calculation (**eigenfrequencies** $_n\omega_k$ **eigenfunctions** $_nu_l(\mathbf{r},\mathbf{t})$)
- Synthetic Seismograms by normal mode summation (k={n,l,m}).



u(**r**,t) Displacement at point **r** at time t due to a force system **F** at point source $\mathbf{r_s}$

$$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k(r) \cos \omega_k t / \omega_k^2 \exp(-\omega_k t/2Q)\, (\mathbf{u_k}.\mathbf{F})_s$$

Source Term $(\mathbf{u_k} .\mathbf{F})_s = (\mathbf{M}:\boldsymbol{\varepsilon})_S$

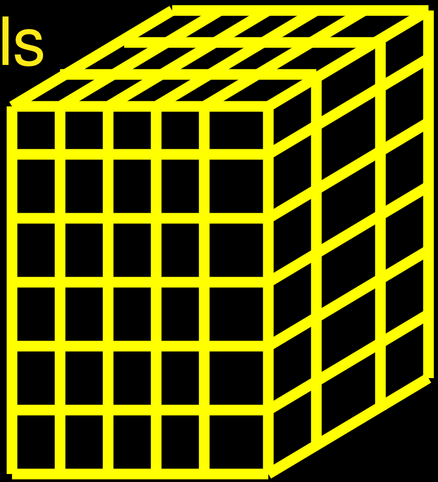
M Seismic moment tensor, **ε** deformation tensor

Parameter Space

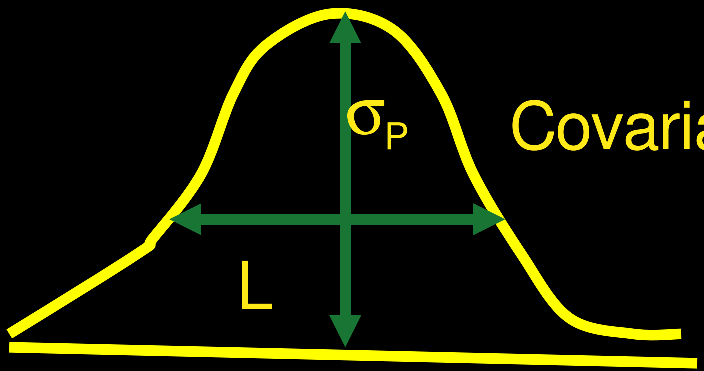
Physical parameters: ρ + 21 (13) physical parameters

Geographical parameterization: $\mathbf{p}(r,\theta,\phi)$

Cells



Continuous parameterization



Covariance fonction

- Spherical harmonic expansion

- Lateral resolution:

Hor 1000km, Rad 50km => 500*60*14 ≈ 420,000 parameters

Différents types de données

-Ondes de volume (temps d’arrivée)
P, S, …..
-Ondes de surface (Rayleigh, Love)
-Modes propres de la terre
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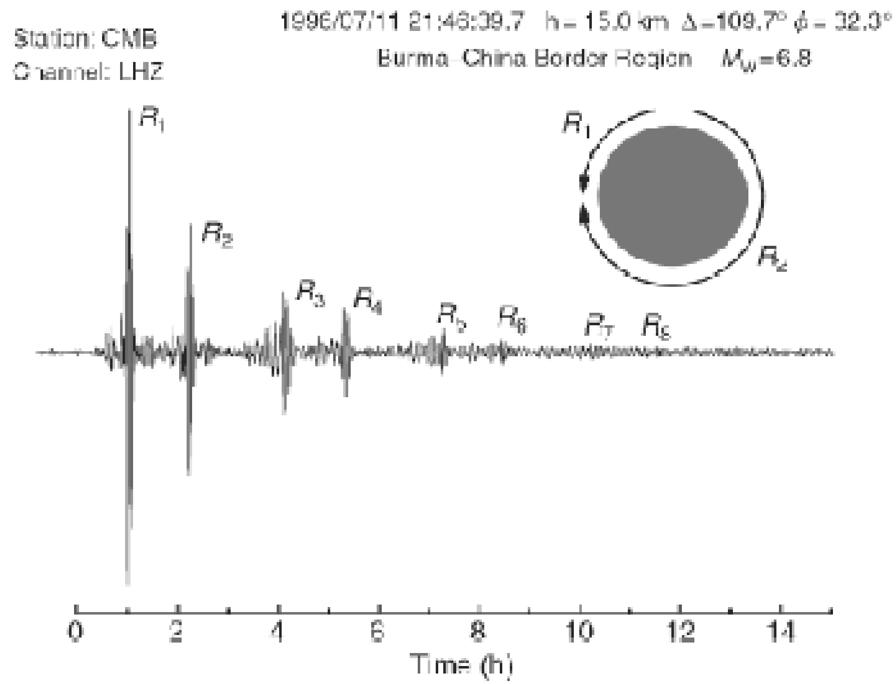
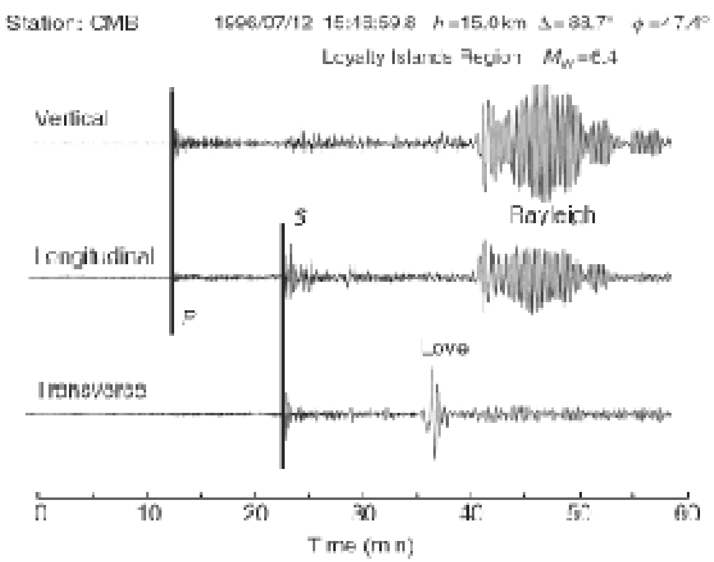
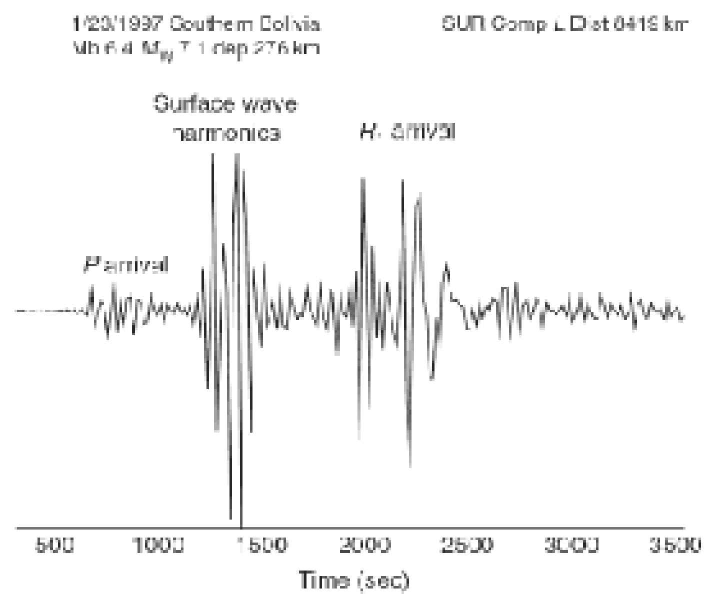
Data







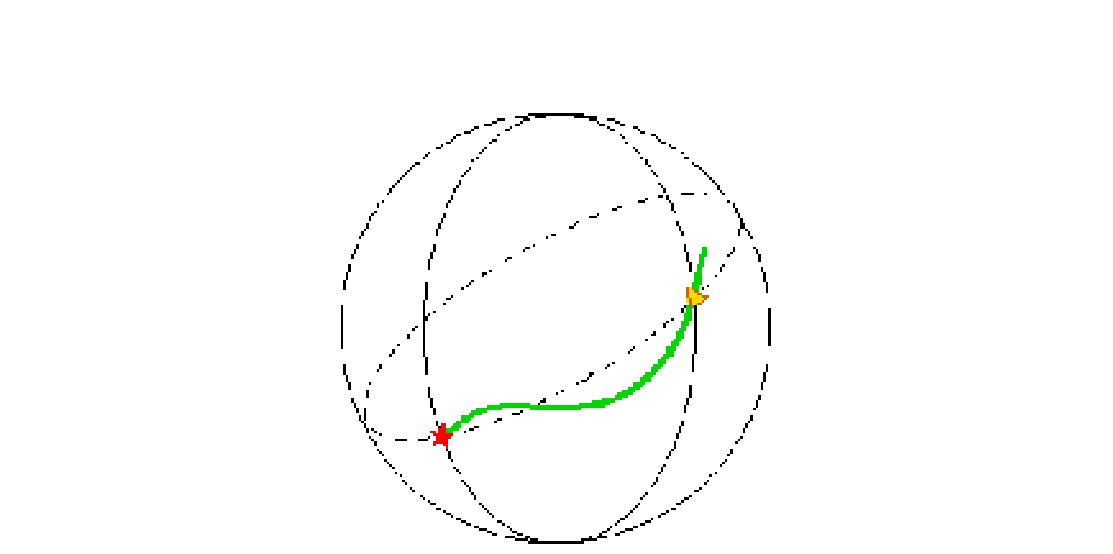
Romanowicz, 2002

Surface wave full ray theory

Waves locally as plane waves $\lambda \ll \Lambda_\Omega$

$$u(\omega) = \sum_{branches} \sum_{orbits} A(\omega) \exp^{-i\phi(\omega)}$$

$$A = A_s A_p A_r \qquad \phi = \phi_s + \phi_p + \phi_r$$



Woodhouse 1974, Woodhouse & Wong 1986, Tromp & Dahlen 1992, Ferreira & Woodhouse 2007a.

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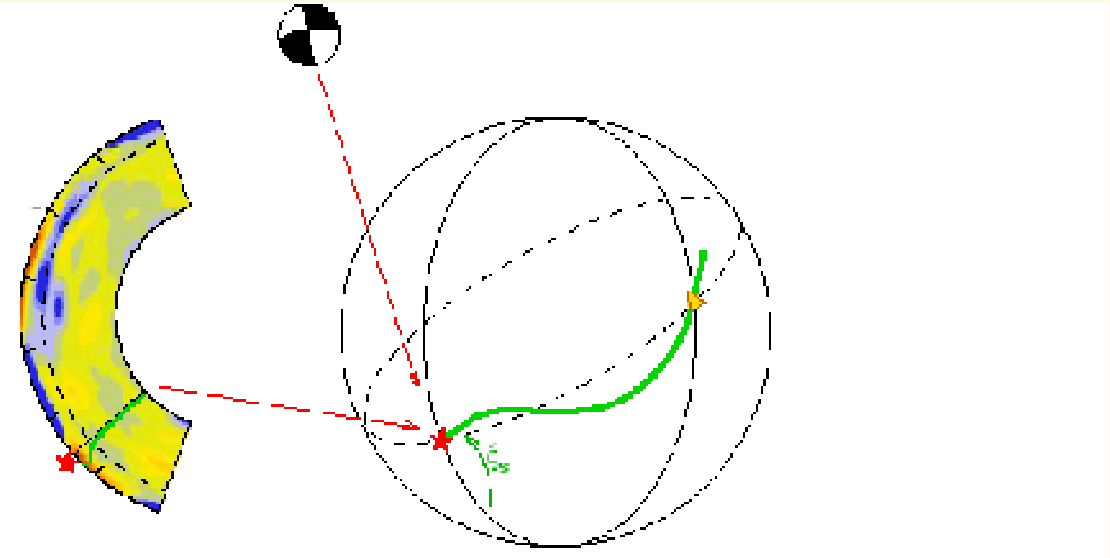

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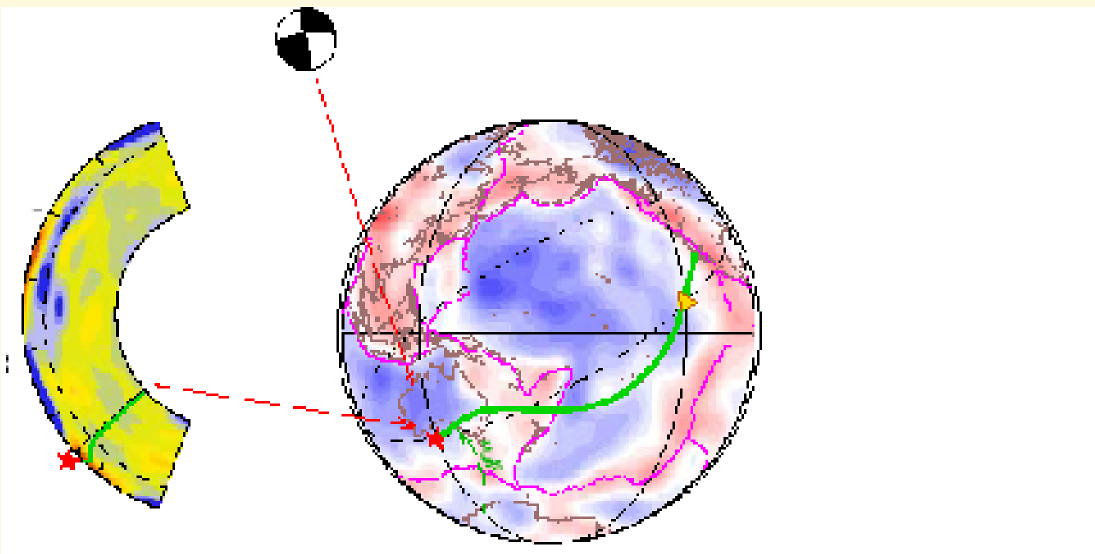

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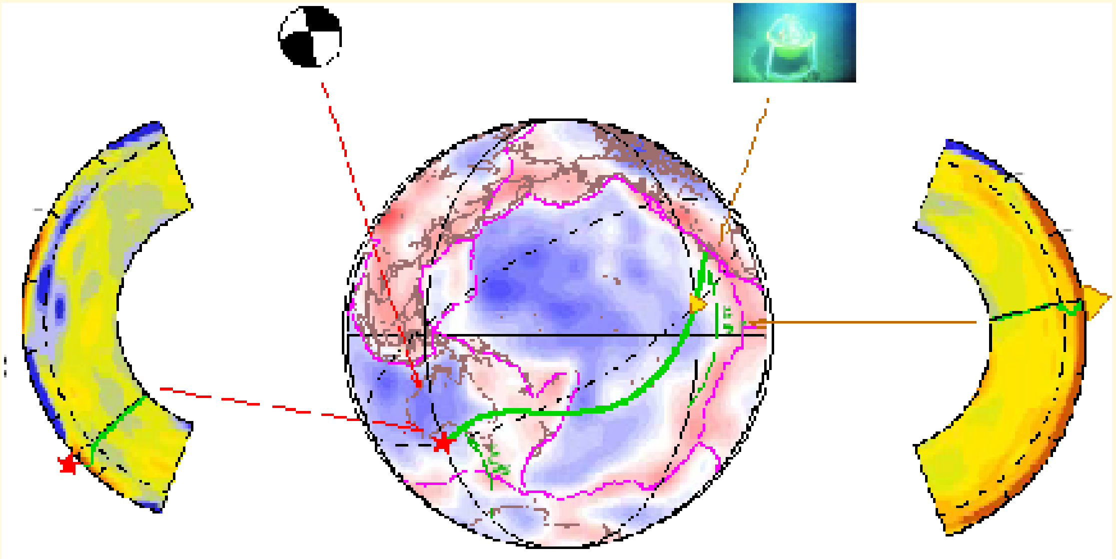


Woodhouse 1974, Woodhouse & Wong 1986, Tromp & Dahlen 1993, Ferreira & Woodhouse 2007a,

Setting up the linearized inverse problem

Phase $\phi - \phi_0 = \omega t - \Delta/v$

Amplitude: source, focalisation, scattering, atténuation

from ray theory(*): $$T = \int_{\mathrm{ray\ path}} \frac{1}{v(\mathbf{r}(s))} \mathrm{d}s$$

Least-squares solution

given a linear inverse problem: $\delta\mathbf{T} = \mathbf{A} \cdot \mathbf{c}$

its least-squares solution:

$$\mathbf{c}_{LS} = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \delta\mathbf{T}$$

is typically solved by either:

direct algorithms (SVD, Cholesky…)

iterative algorithms (CG, LSQR…)

Setting up the linearized inverse problem

from ray theory: $$T = \int_{\text{ray path}} \frac{1}{v(\mathbf{r}(s))} \mathrm{d}s$$

first-order Taylor expansion: $$\delta T = -\int_{\text{ray path}} \frac{\delta v(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

"parameterization": $$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

Unknown coefficients are constant and don't need to be

$$\delta T = -\sum_{i=1}^{N} c_i \int_{\text{ray path}} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

Setting up the linearized inverse problem

$$\delta T = -\sum_{i=1}^{N} c_i \int_{\text{ray path}} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

The same eq. Applies for many data and corresp. ray paths:

$$\delta T_j = -\sum_{i=1}^{N} c_i \int_{\text{ray path}_j} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s \qquad (j{=}1,\ldots,M)$$

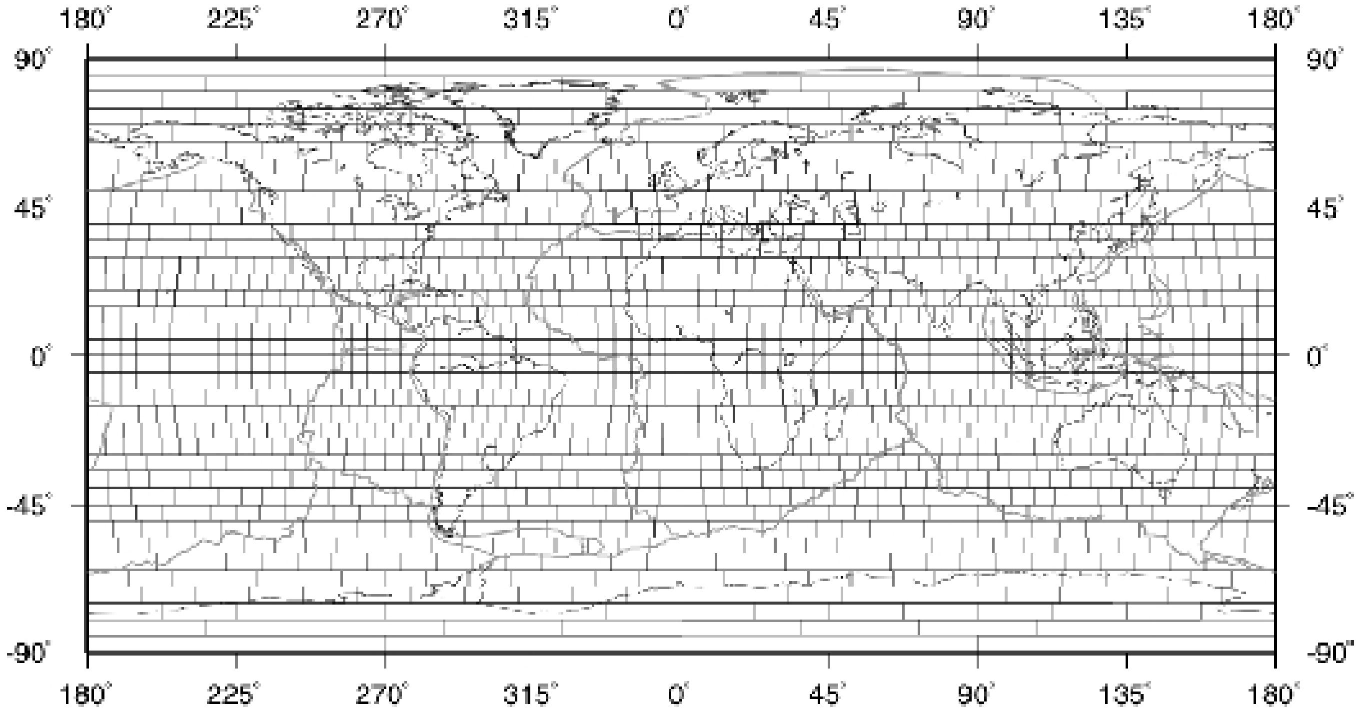
given a linear inverse problem: $\delta T = G\ \delta v$

$$\delta \mathbf{T} = \mathbf{A} \cdot \mathbf{c}$$

Parameterization, or choice of basis functions

$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

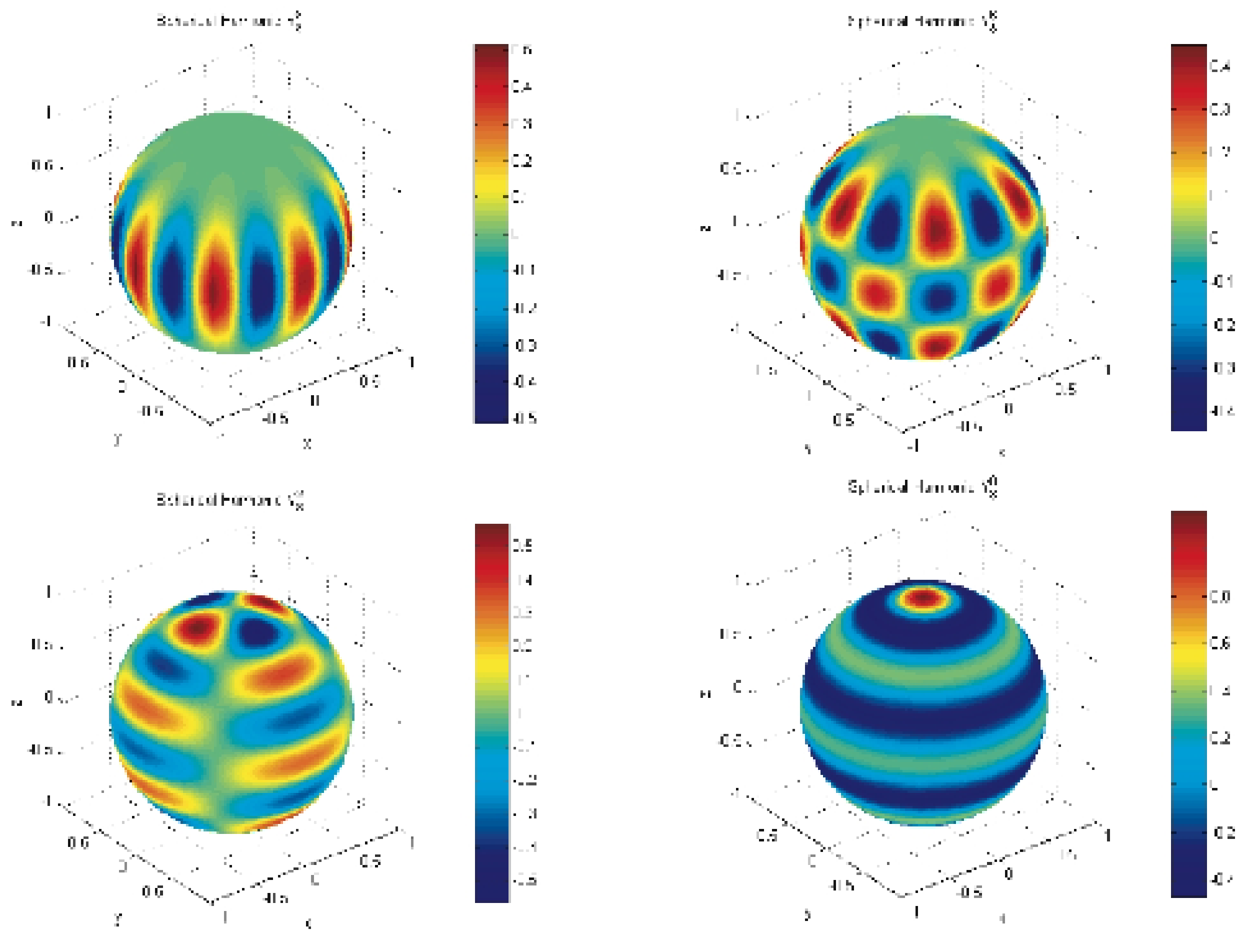
pixels ("local" basis functions)



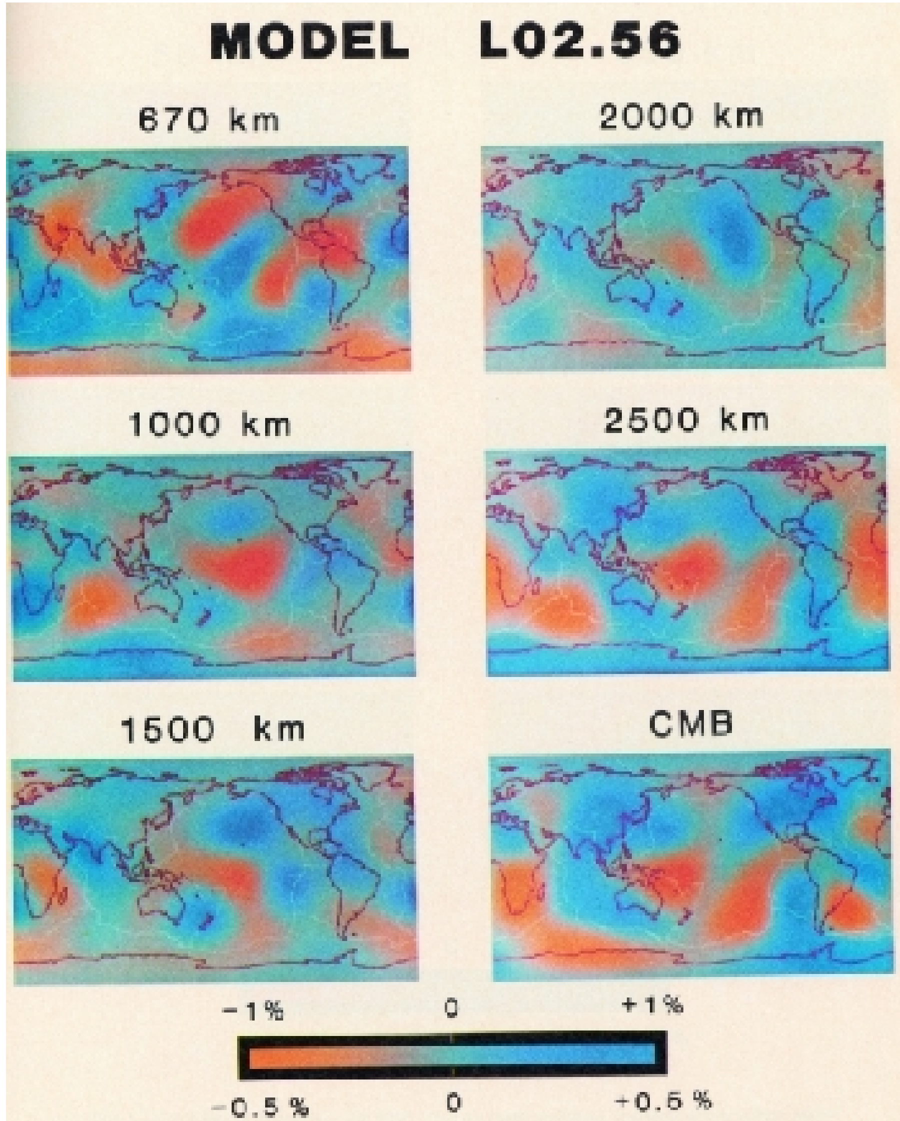
Parameterization, or choice of basis functions

$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

spherical harmonics
(“global” basis
functions)

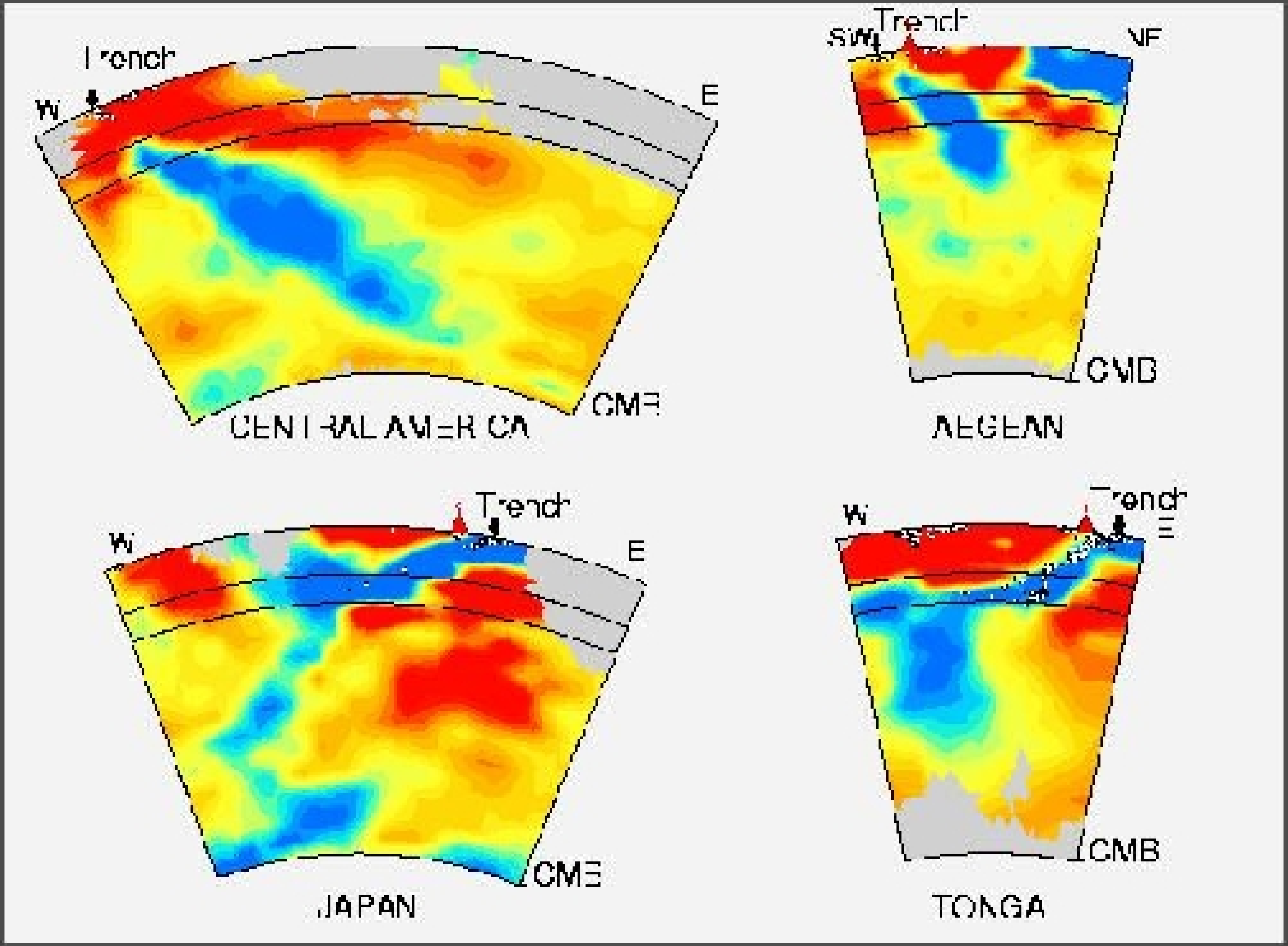


body wave tomography: examples



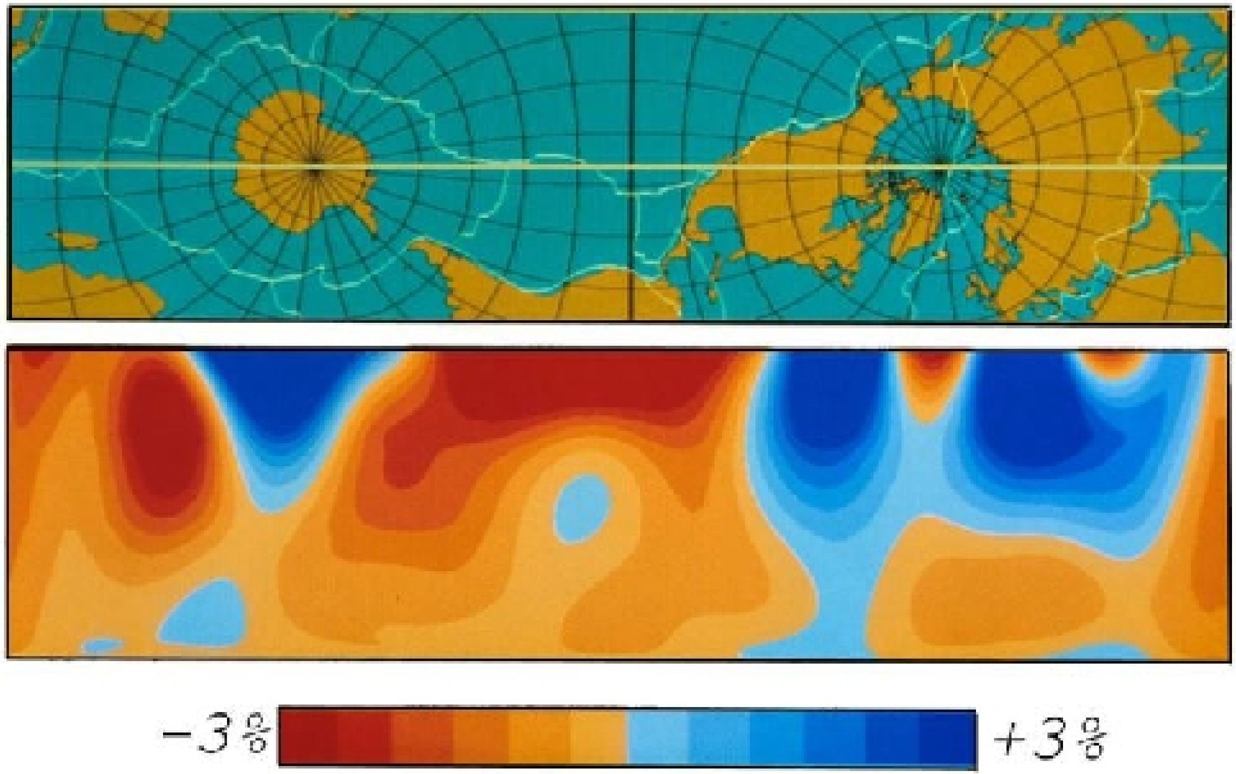
1980s low-degree model (Dziewonski)

Imaging of Slabs



Hilst et al., 1997

body wave (+surface wave) tomography: examples



1980s low-degree model (Dziewonski & Woodhouse), cross-sec

Tomographie d'onde de surface

Phase $\phi - \phi_0 = \omega t - \Delta/c$

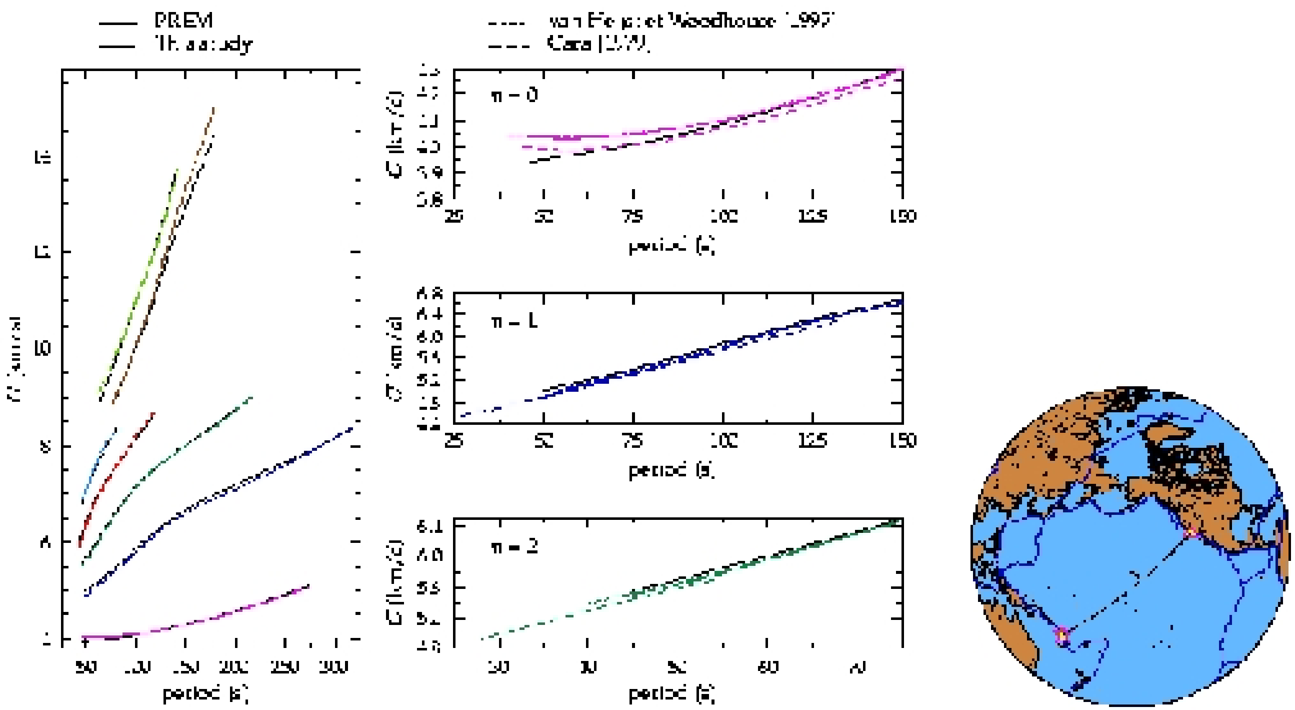
C(T) phase velocity, C_0(T) ref. phase velocity

What tomographers need is the relationship between displacement **u** and Earth's "structure", $\rho(\mathbf{r})\ \lambda(\mathbf{r})\ \mu(\mathbf{r})$

we can *linearize* it in a perturbative approach:

$$\delta\mathbf{u} = \int_V \left[\mathbf{K}_\rho(\mathbf{r})\delta\rho(\mathbf{r}) + \mathbf{K}_\lambda(\mathbf{r})\delta\lambda(\mathbf{r}) + \mathbf{K}_\mu(\mathbf{r})\delta\mu(\mathbf{r})\right] \mathrm{d}V$$

1st step: Calculation of dispersion curves: Fundamental modes and higher modes

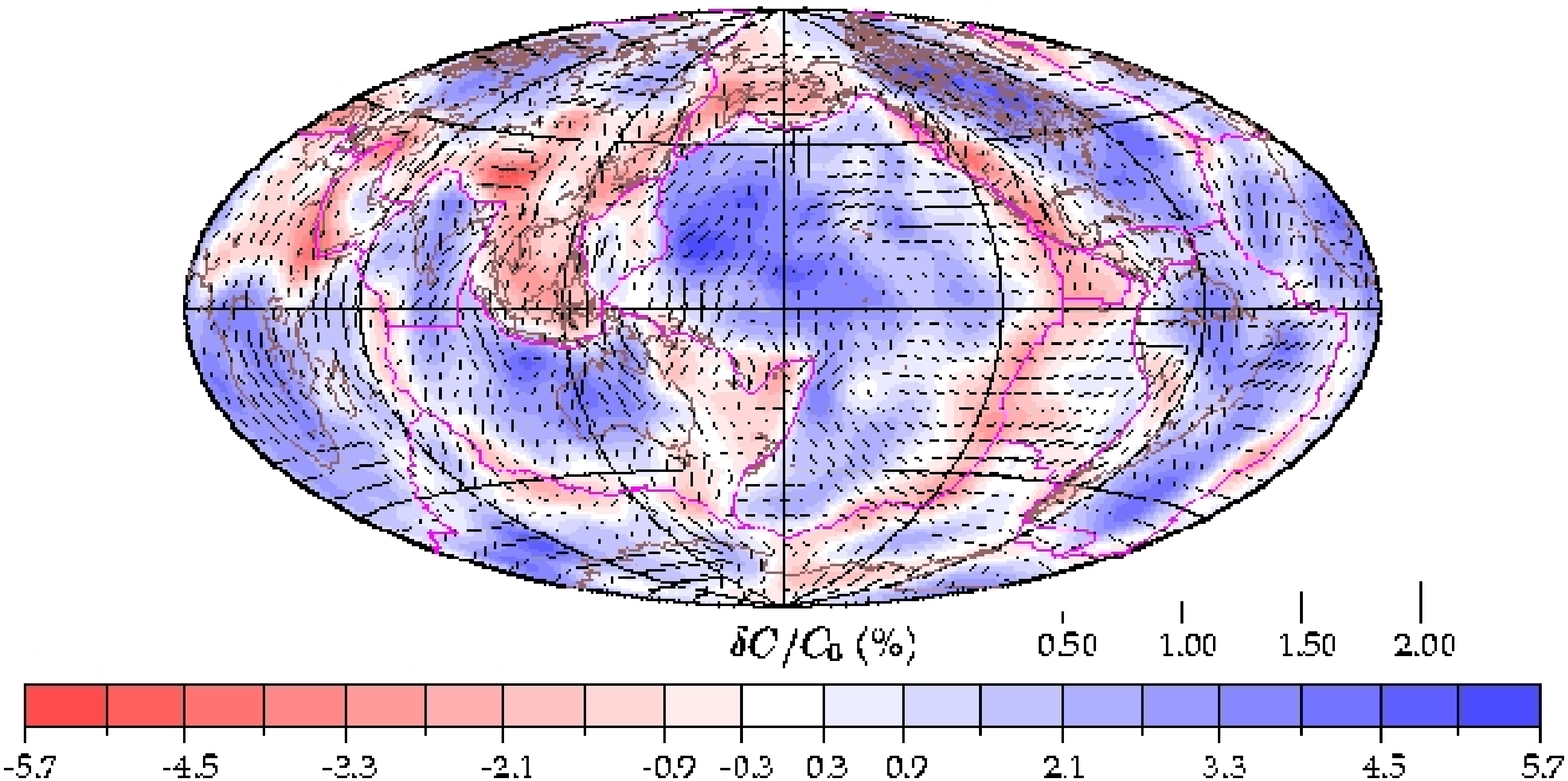


Comparison with previous results along the Vanuatu-California path.

Beucler et al., 2002

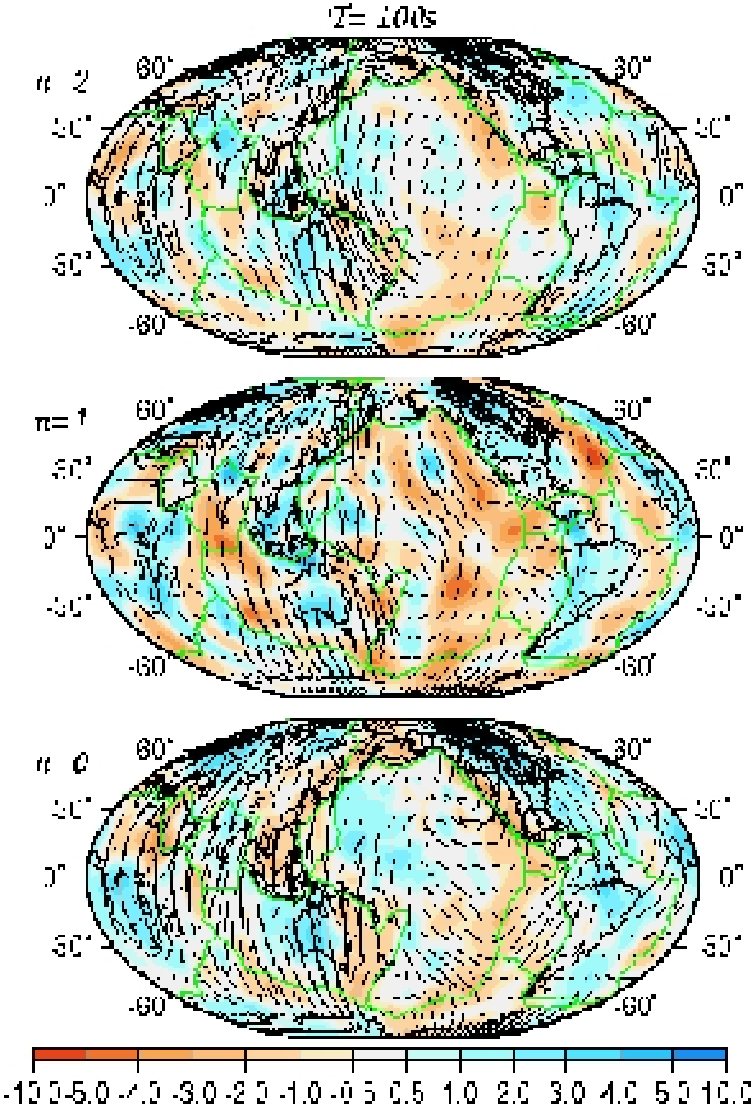
2nd step: regionalization

ÉRIC BEUCLER 2002



Anisotropic model resulting from phase velocities inversion (C.L.A.S.H., including 2Ψ and 4Ψ terms), $n = 0$, $T = 50$ s.

Phase velocity maps
At 100s



2nd overtone

1st overtone

Fundamental mode

Beucler and Montagner, 2006

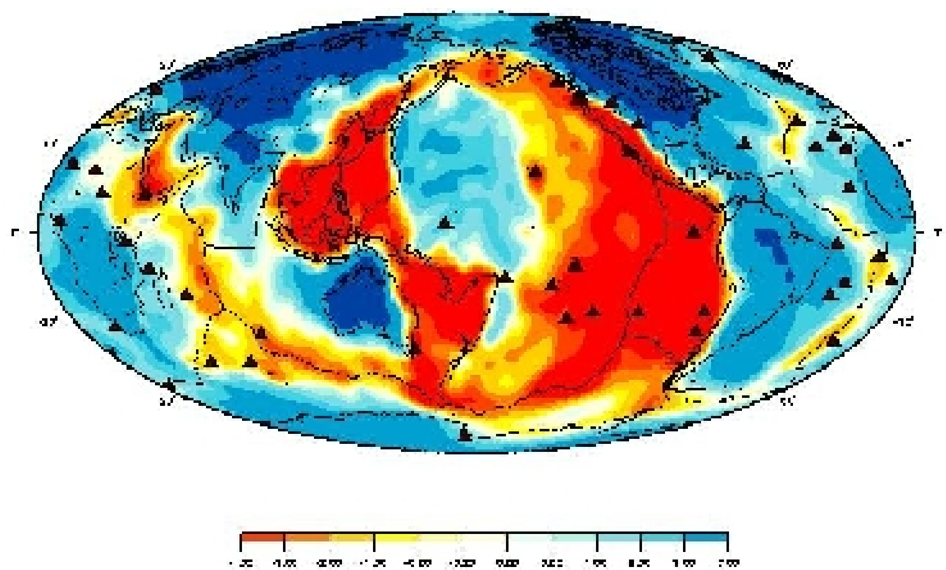
Global Tomography
3rd step: Inversion at depth

Scale $\Lambda \approx$ 2000km (degree 20)

Seismic wavelength $\lambda \leq$ 500km

Ray theory applies

Shear wave velocities - depth = 100km

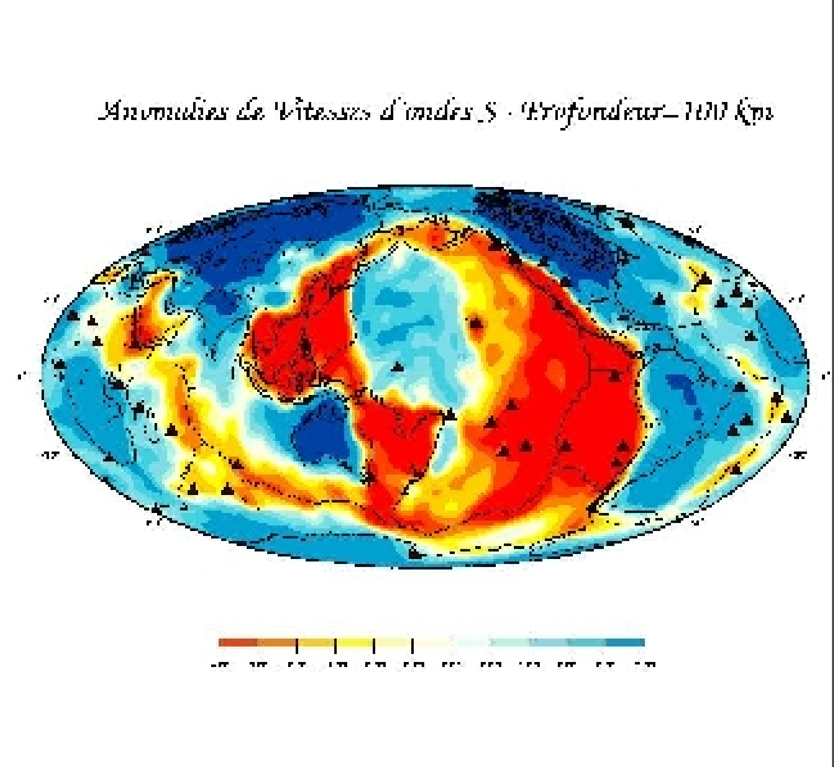


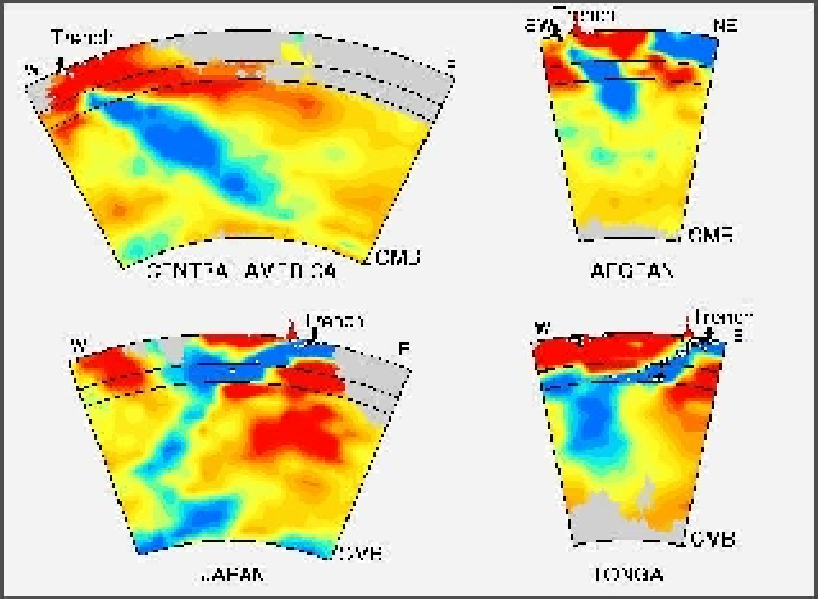
Global and Regional Tomography

From Global Scale to Regional scale

Scale $\Lambda \approx$ 200-500km

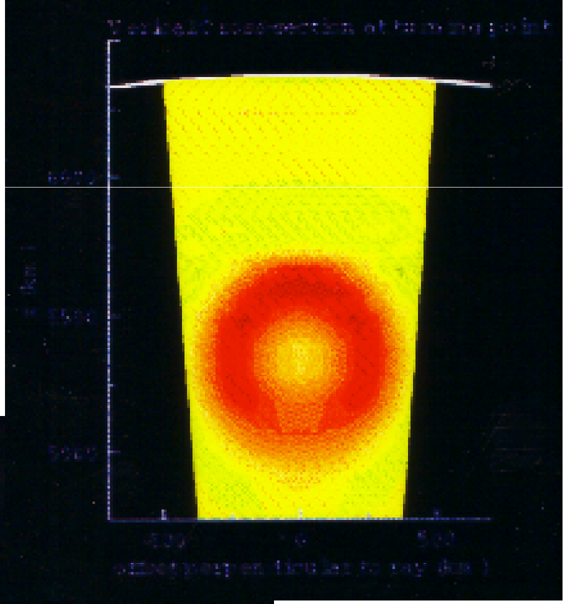
Seismic wavelength 20km $\leq \lambda \leq$ 500km

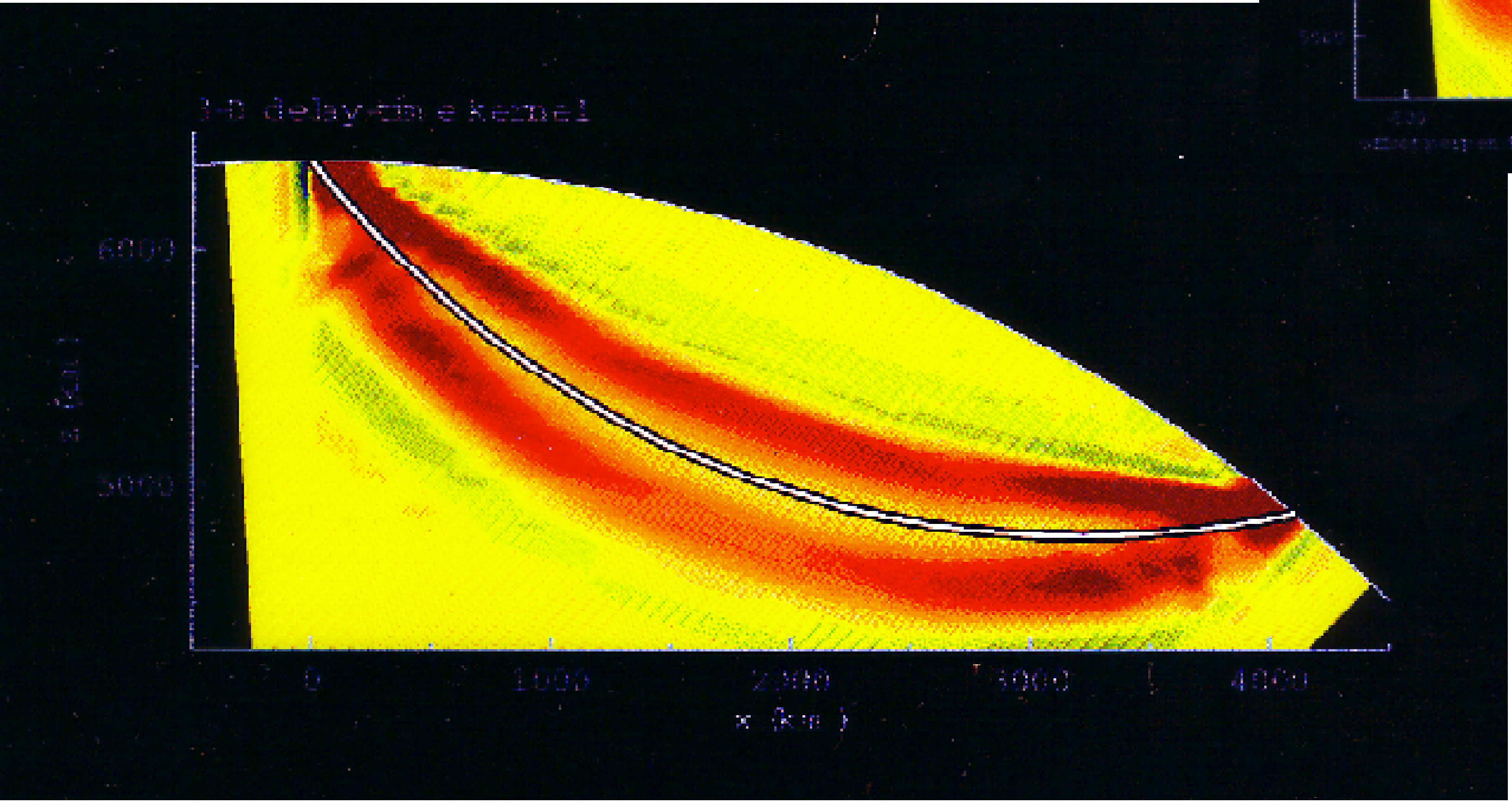




Van der Hilst et al., 1998

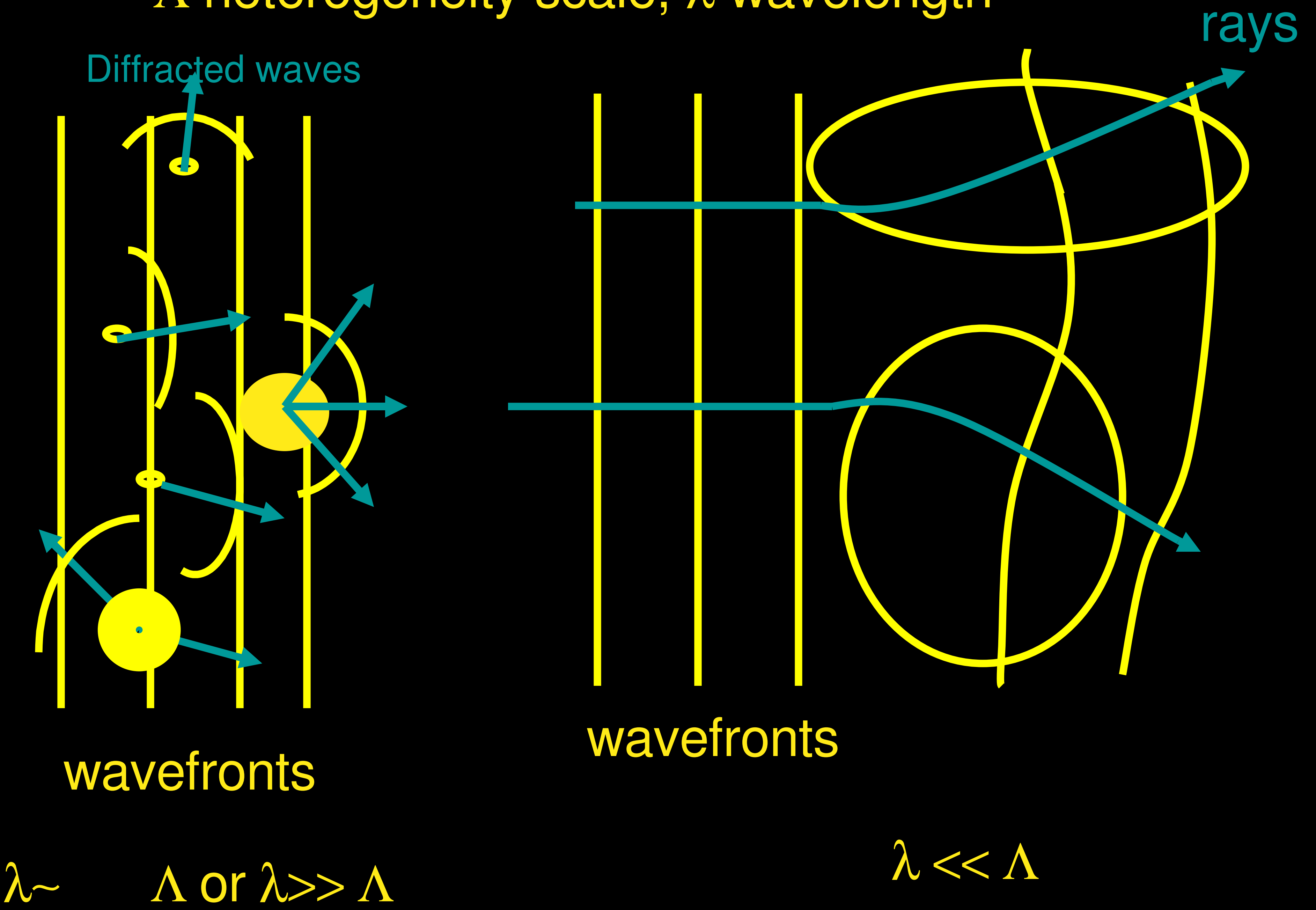
Sensitivity of the delay time to the local seismic velocity





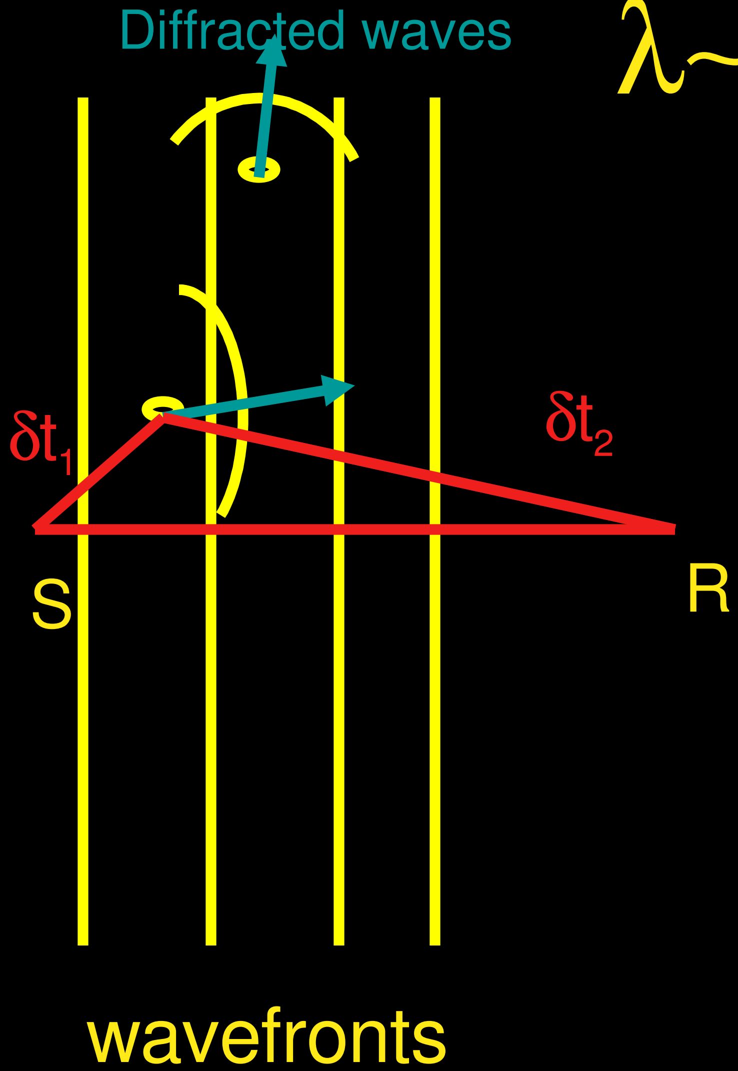
Marquering et al., GJI 1998

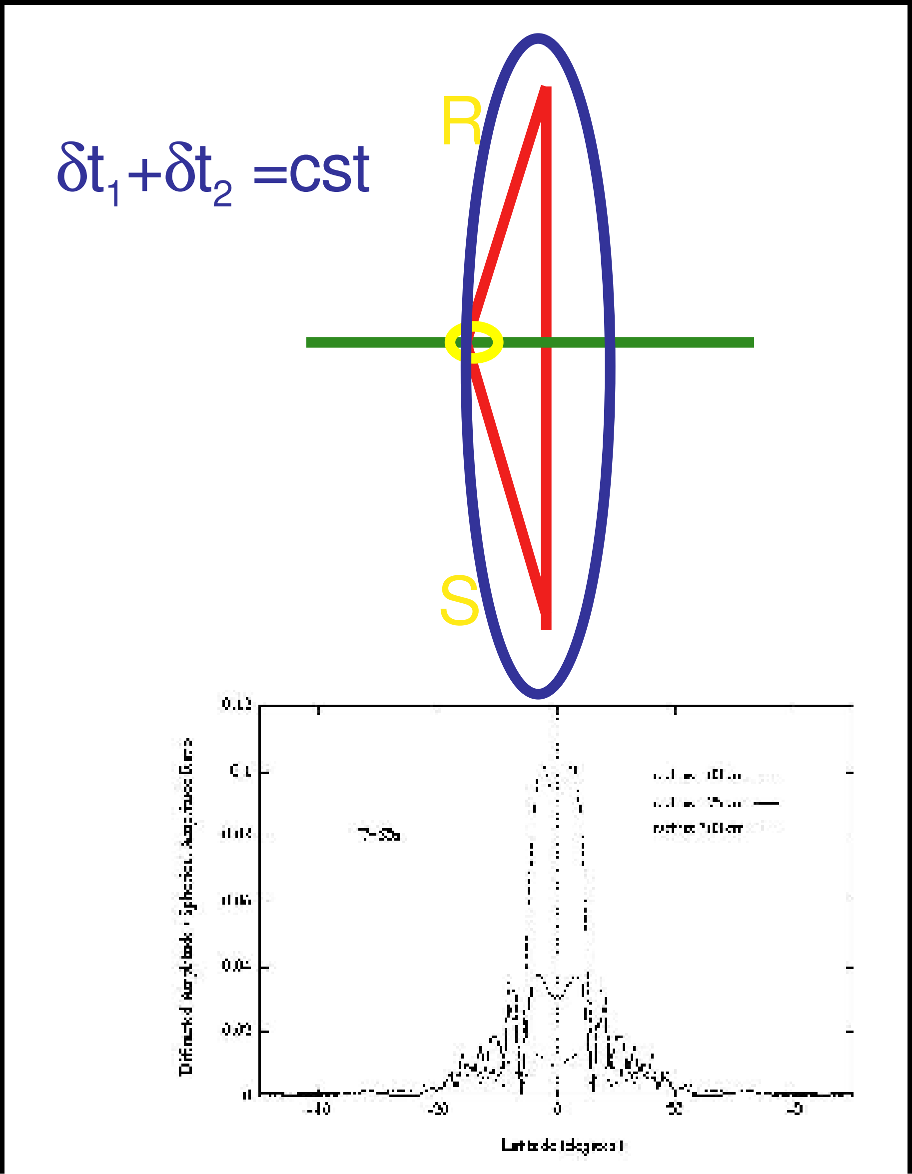
Λ heterogeneity scale, λ wavelength



Λ heterogeneity scale, λ wavelength

$\lambda \sim \Lambda$ or $\lambda \gg \Lambda$





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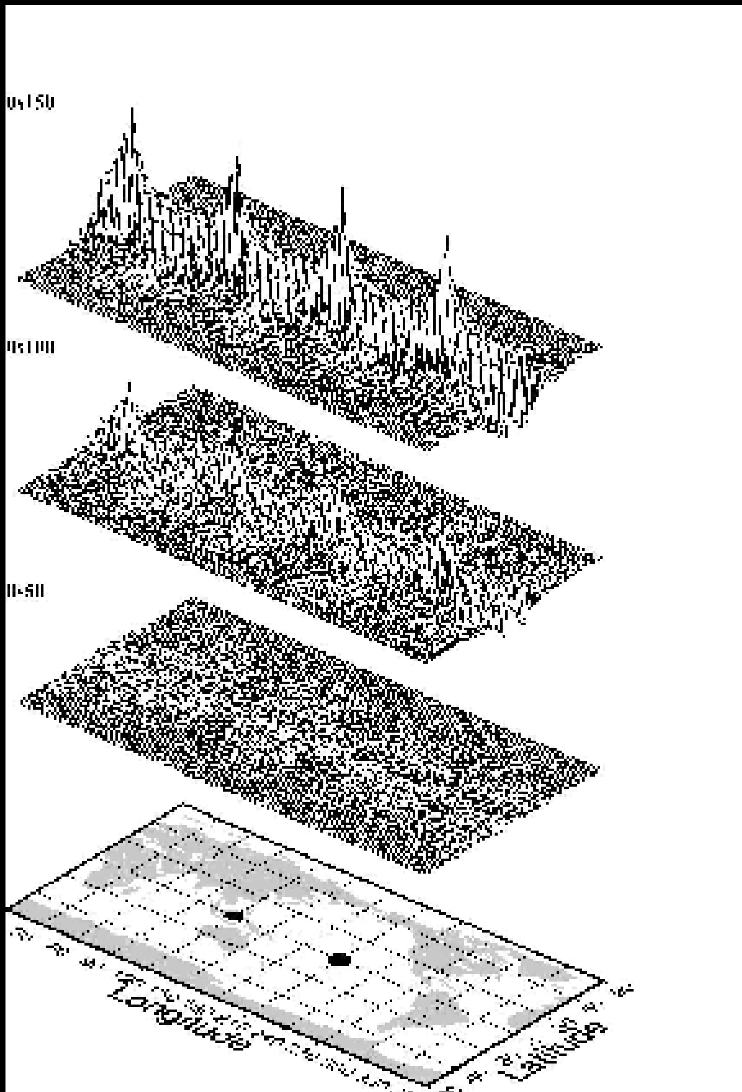
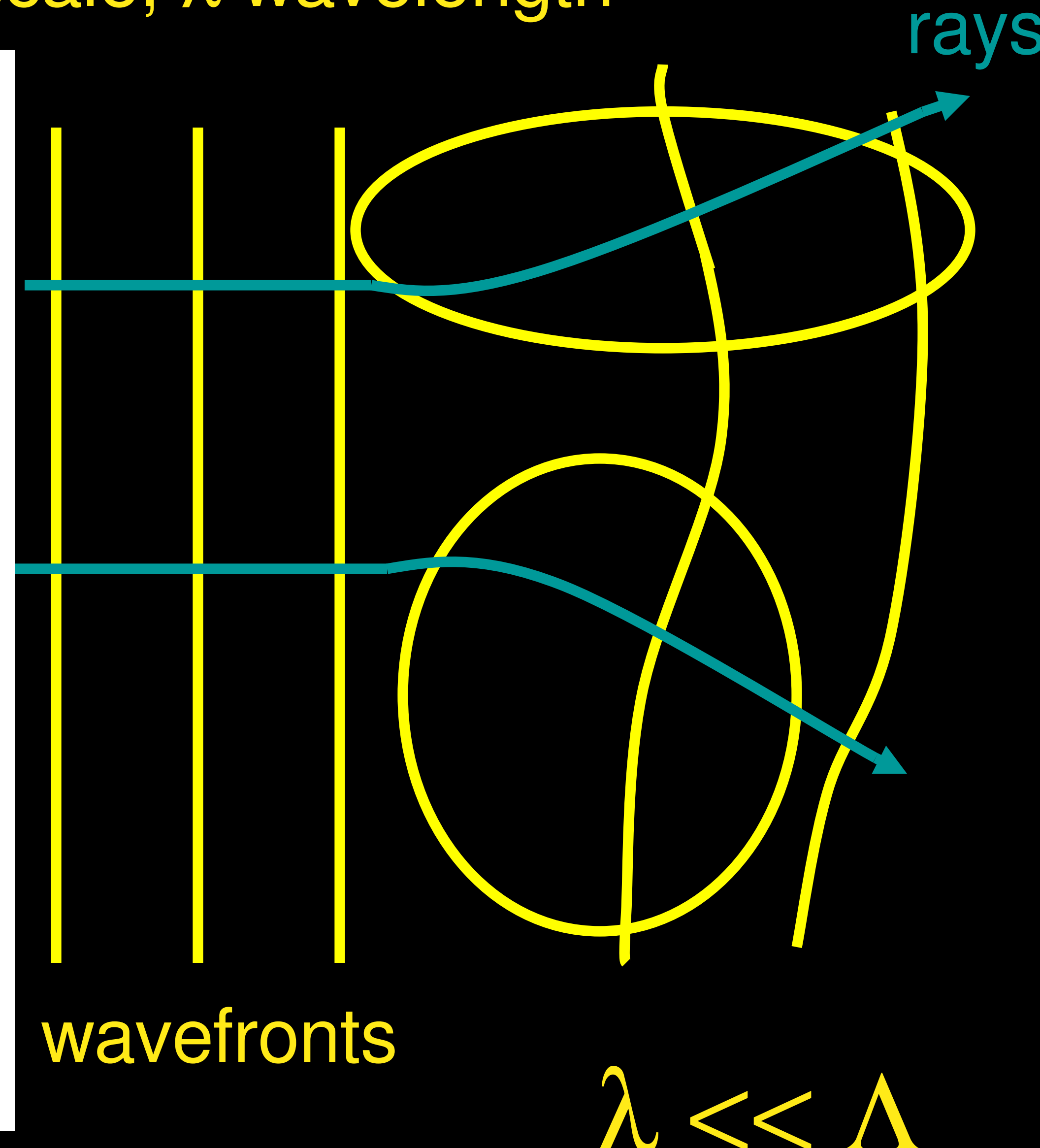
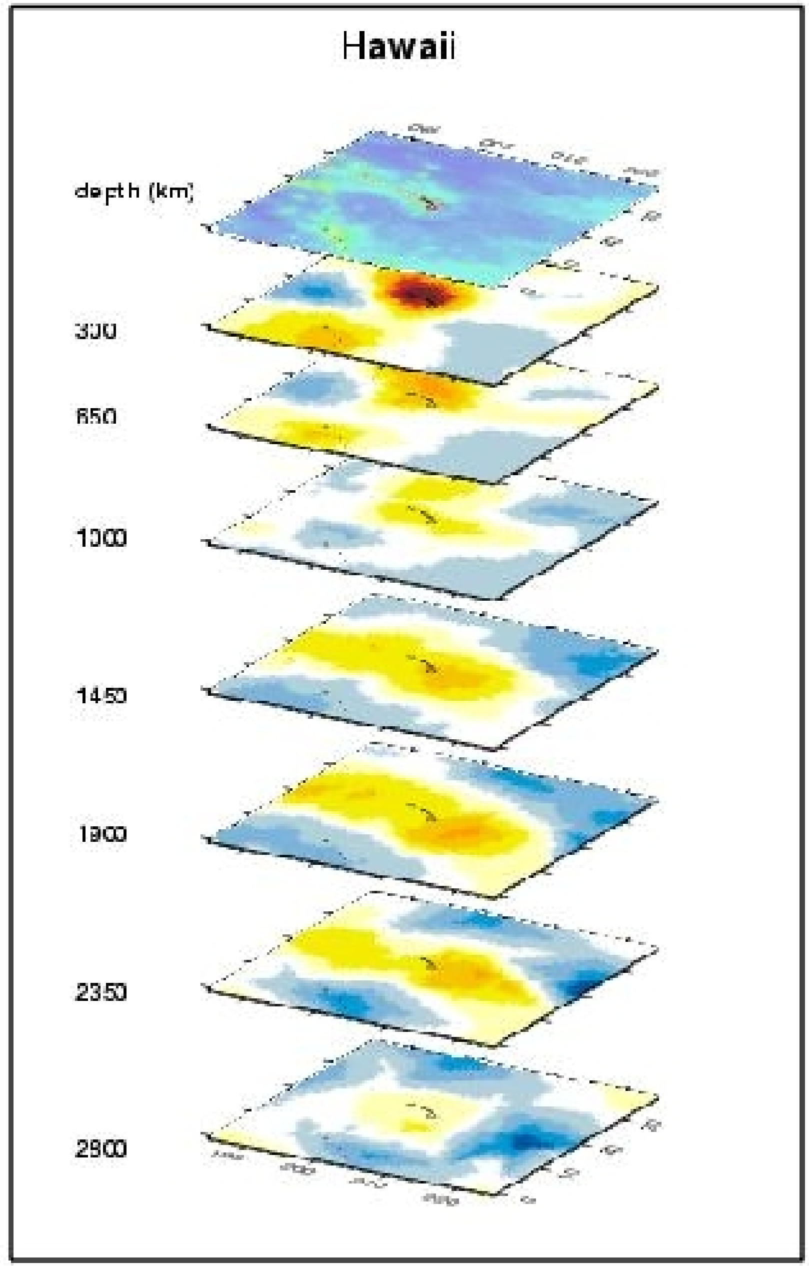




Typical scales Λ≈2,000km, λ≈ 500km

Banana-Doughnut Theory (Dahlen et al.)

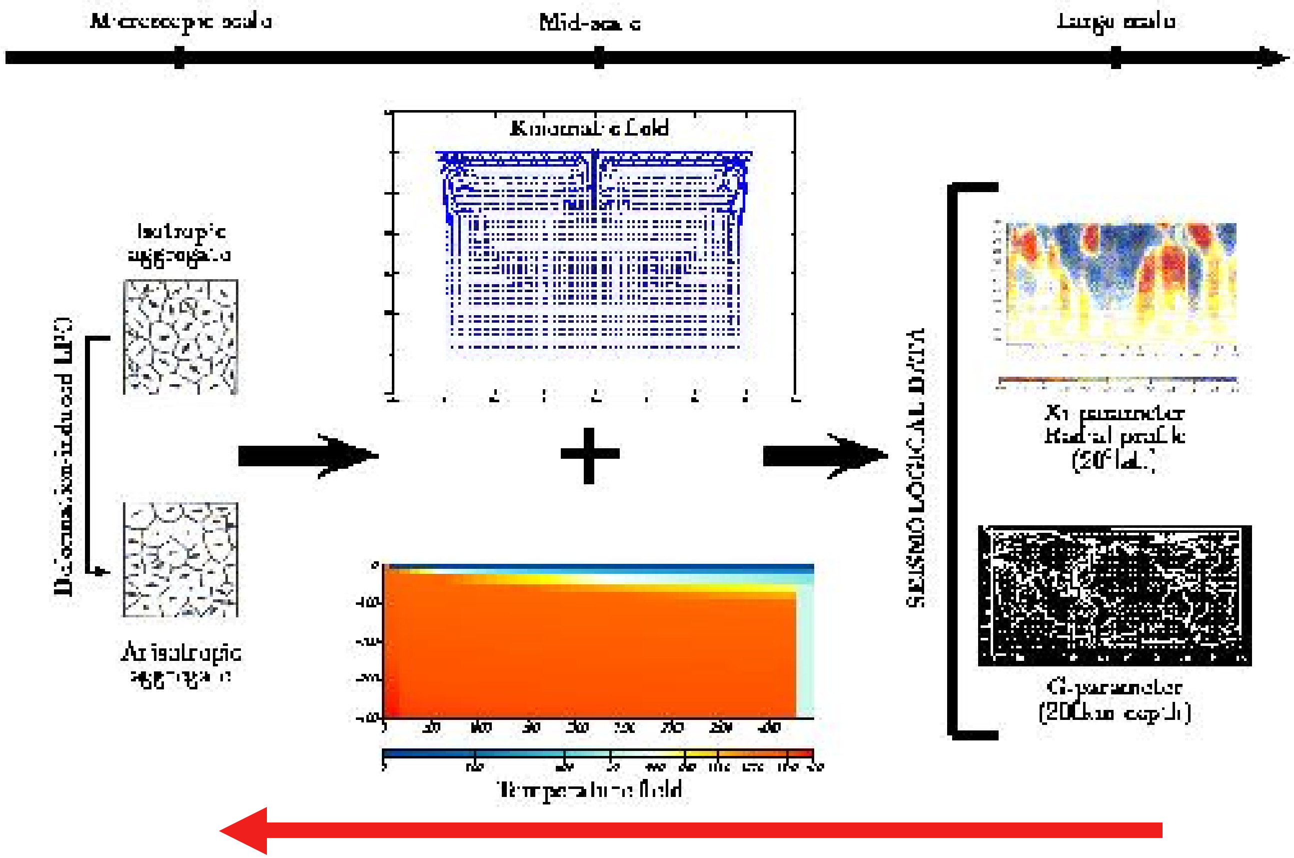
Application to global tomography (Montelli et al., Science, 2004)



Importance of seismic anisotropy

ANISOTROPY is the Rule not the Exception

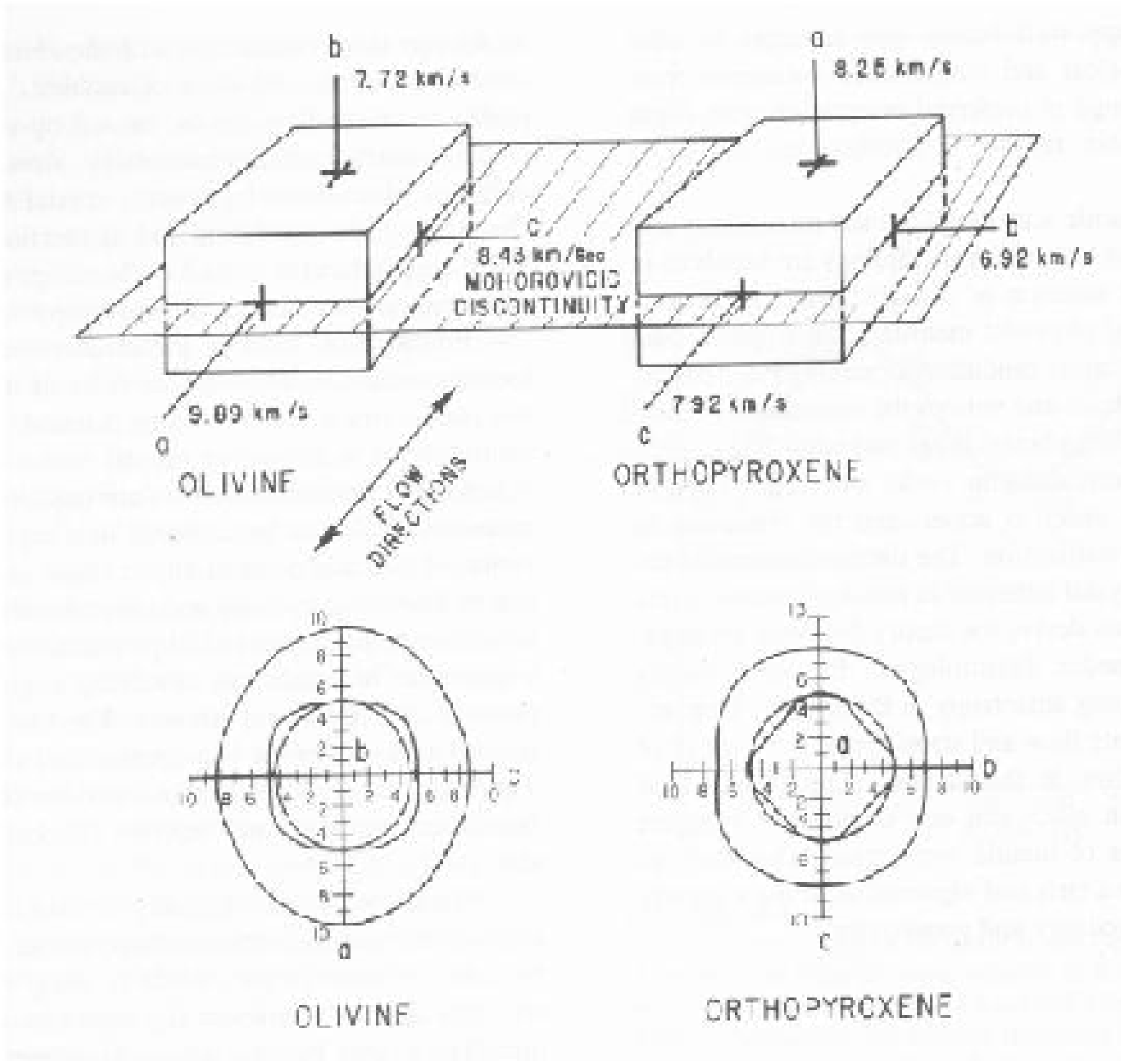
Seismic Anisotropy is present at all scales



Mineralogical composition

Mapping Convection

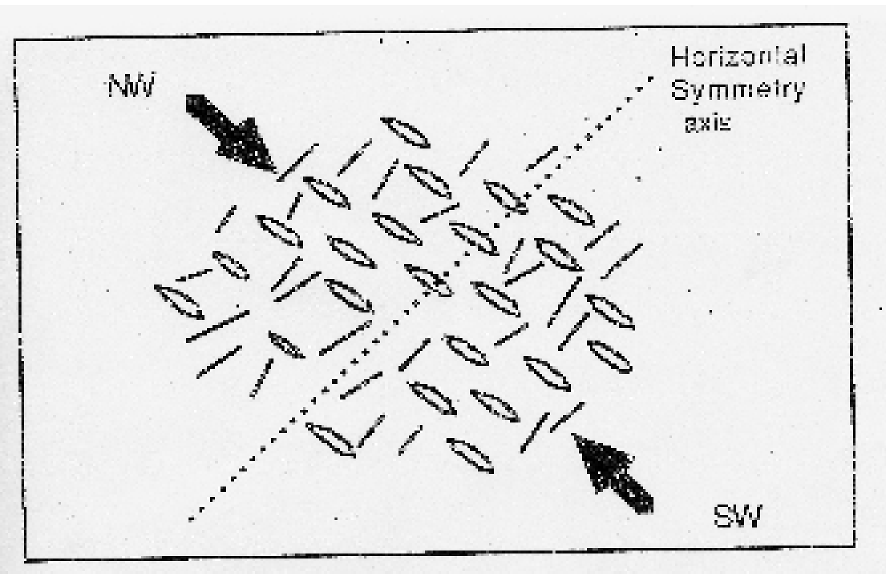
(Montagner and Guillot, 2001)



From Christensen and Lundquist, 1982

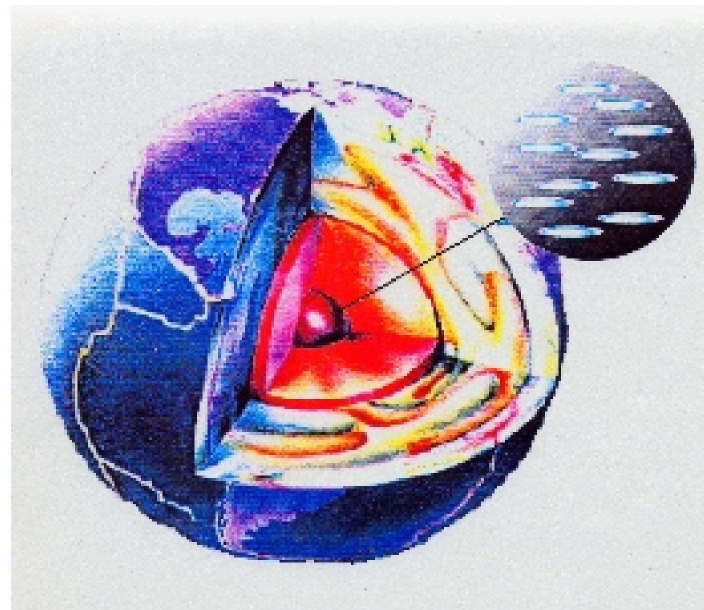
Cracks, fluid inclusions

Crust



(Babuska and Cara, 1991)

Inner core



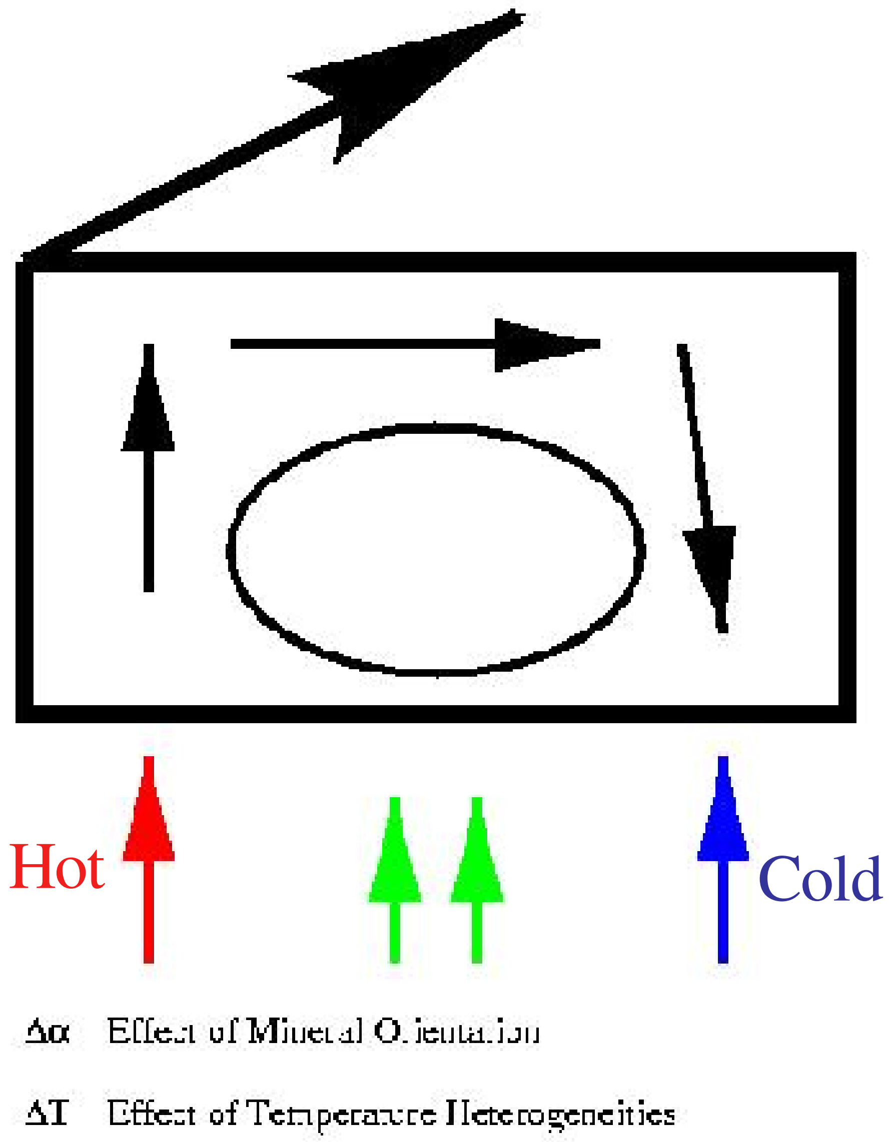
(Singh et al., 2001)

Importance of seismic anisotropy
ANISOTROPY is the Rule not the Exception

Anisotropy is present at all scales

-From microscopic scale up to macroscopic scale
-Efficient mechanisms of alignment (L.P.O.: lattice preferred orientation S.P.O.: shape preferred orientation; fine layering)

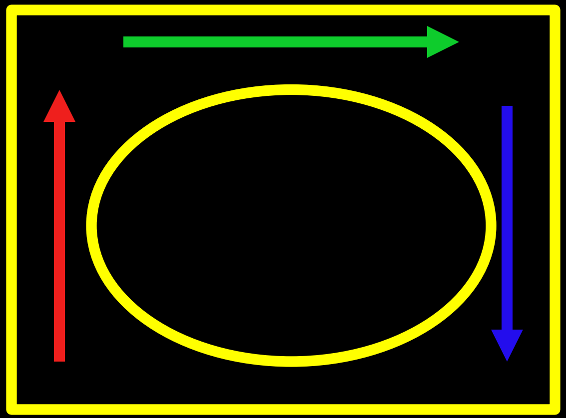
Montagner &Guillot, 2001



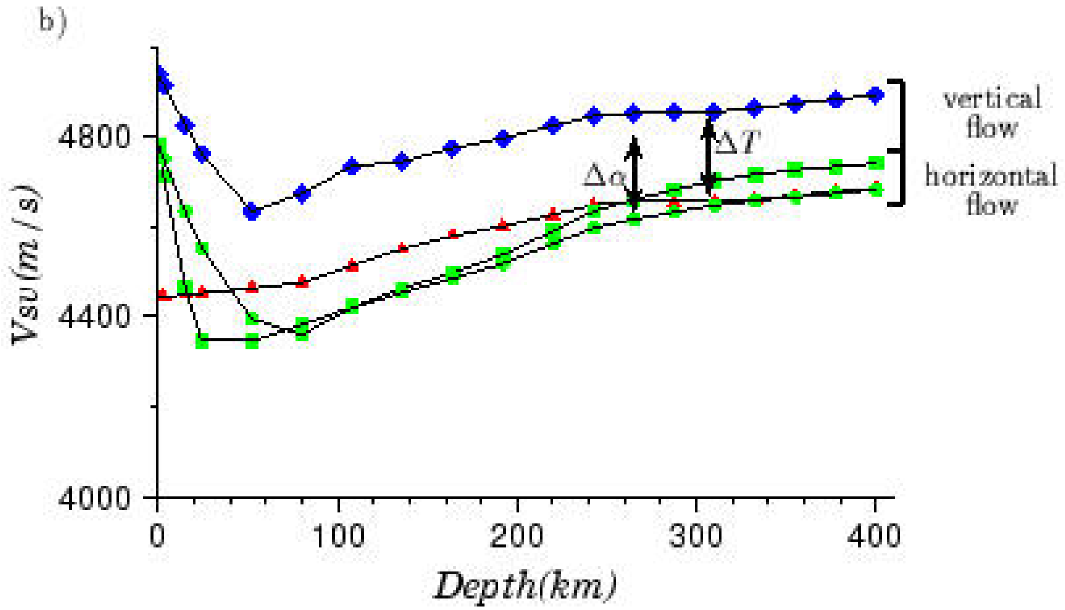
$\Delta\alpha$:Anisotropy Effect

ΔT:Temperature Effect

$$\Delta\alpha \approx \Delta T$$



Olivine (60%) +Opx (40%)

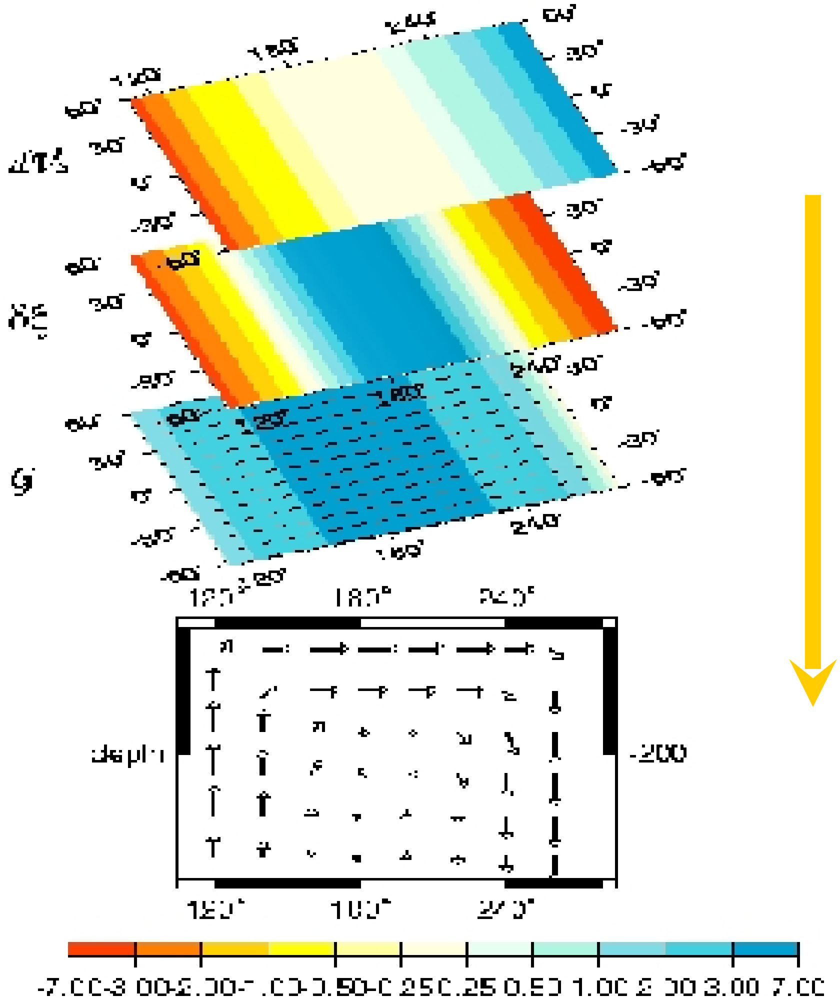


Montagner & Guillot, 2002

S- Velocity

Radial Anisotropy

Azimuthal Anisotropy



Interpretation

Anisotropy is observed on different kinds of seismic waves

- Body waves (Pn, Shear wave splitting)
- Surface waves (Rayleigh-Love discrepancy; Azimuthal anisotropy)

Anisotropy is present at all scales

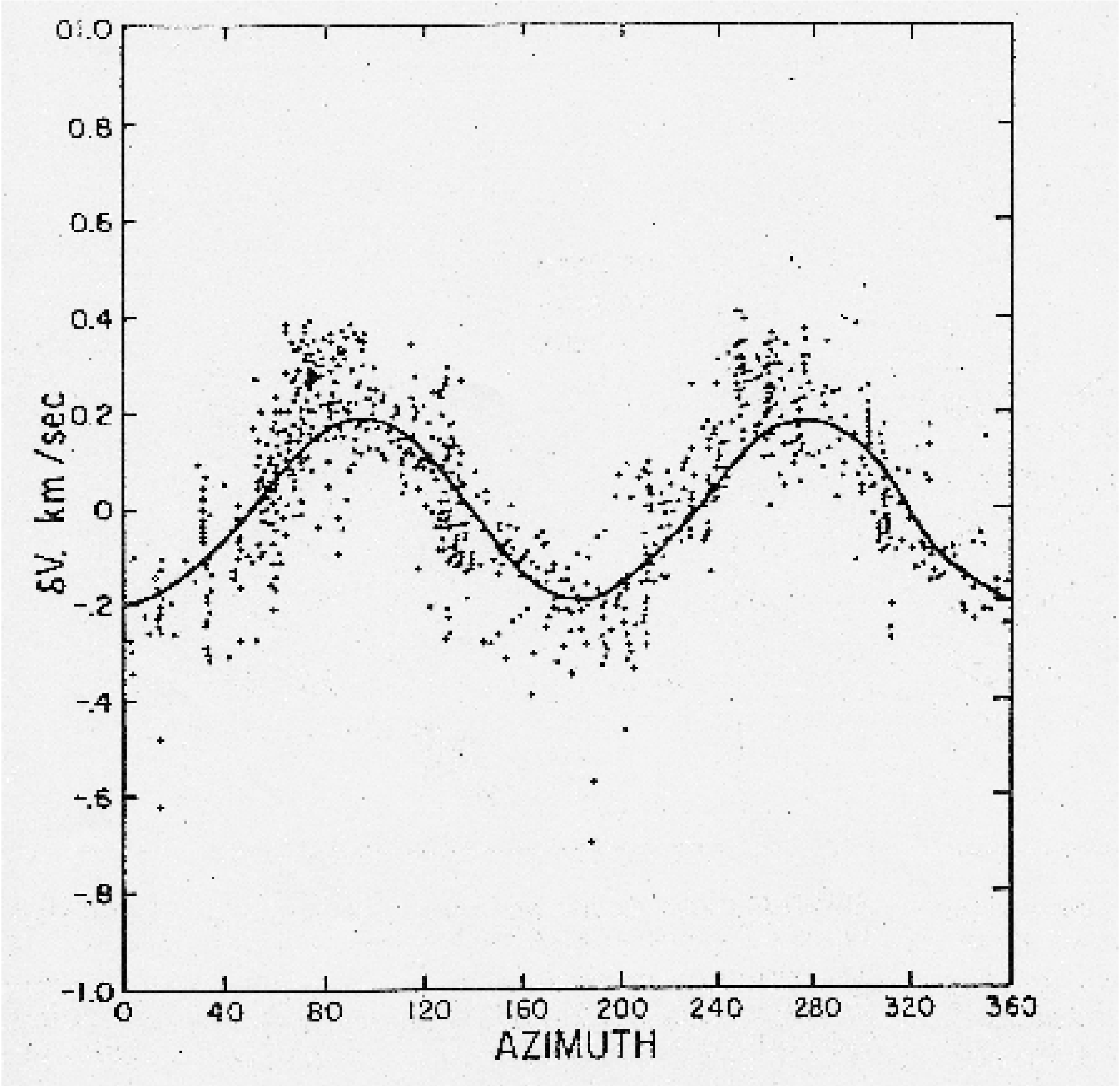
- from microscopic scale to macroscopic scale
-Efficient mechanisms of alignment
(L.P.O.: lattice preferred orientation
S.P.O.: shape preferred orientation; fine layering)

Anisotropy is observed on different kinds of seismic waves
-Body waves (Pn; S-wave splitting)
-Surface waves (Rayleigh-Love discrepancy, azimuthal anisotropy)

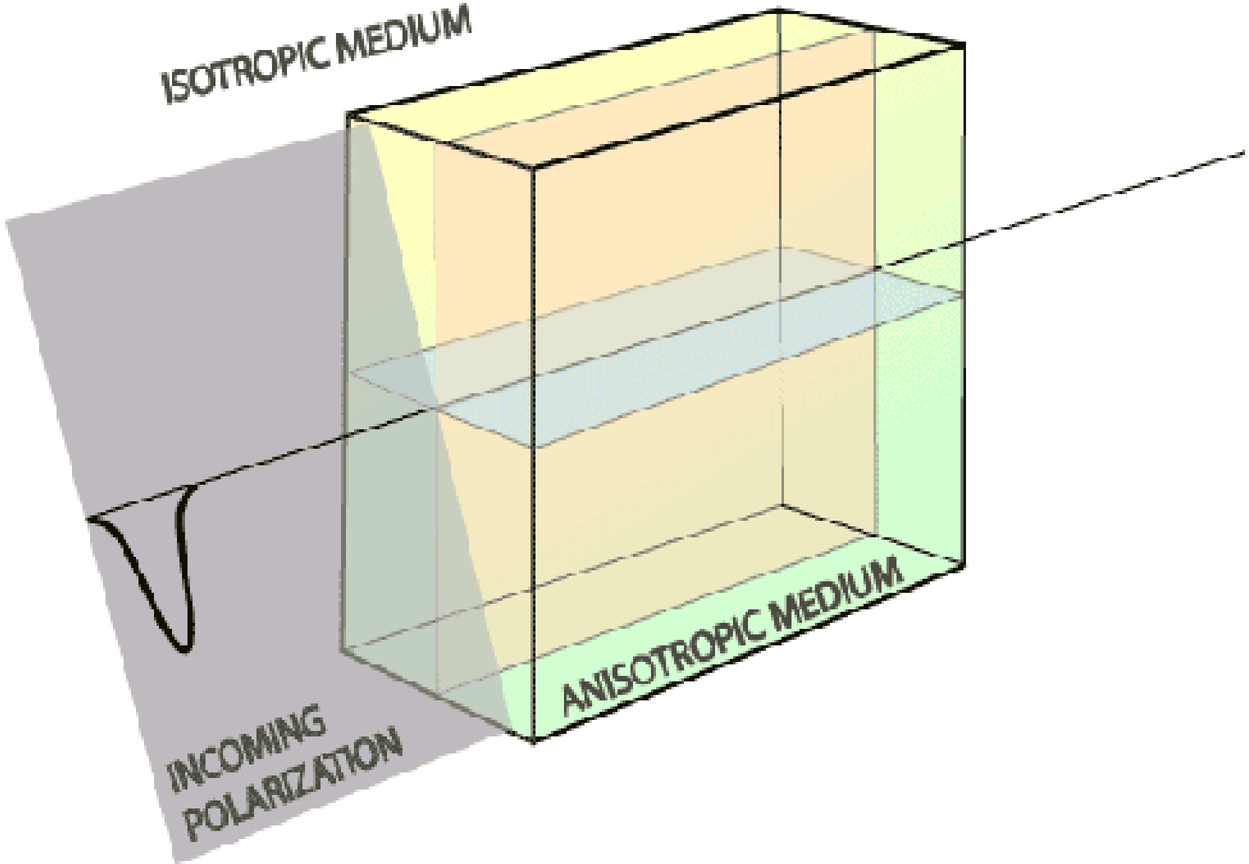
ANISOTROPY REFLECTS AN INNER ORGANIZATION

ANISOTROPY IS NOT A SECOND ORDER EFFECT

Pn- velocities

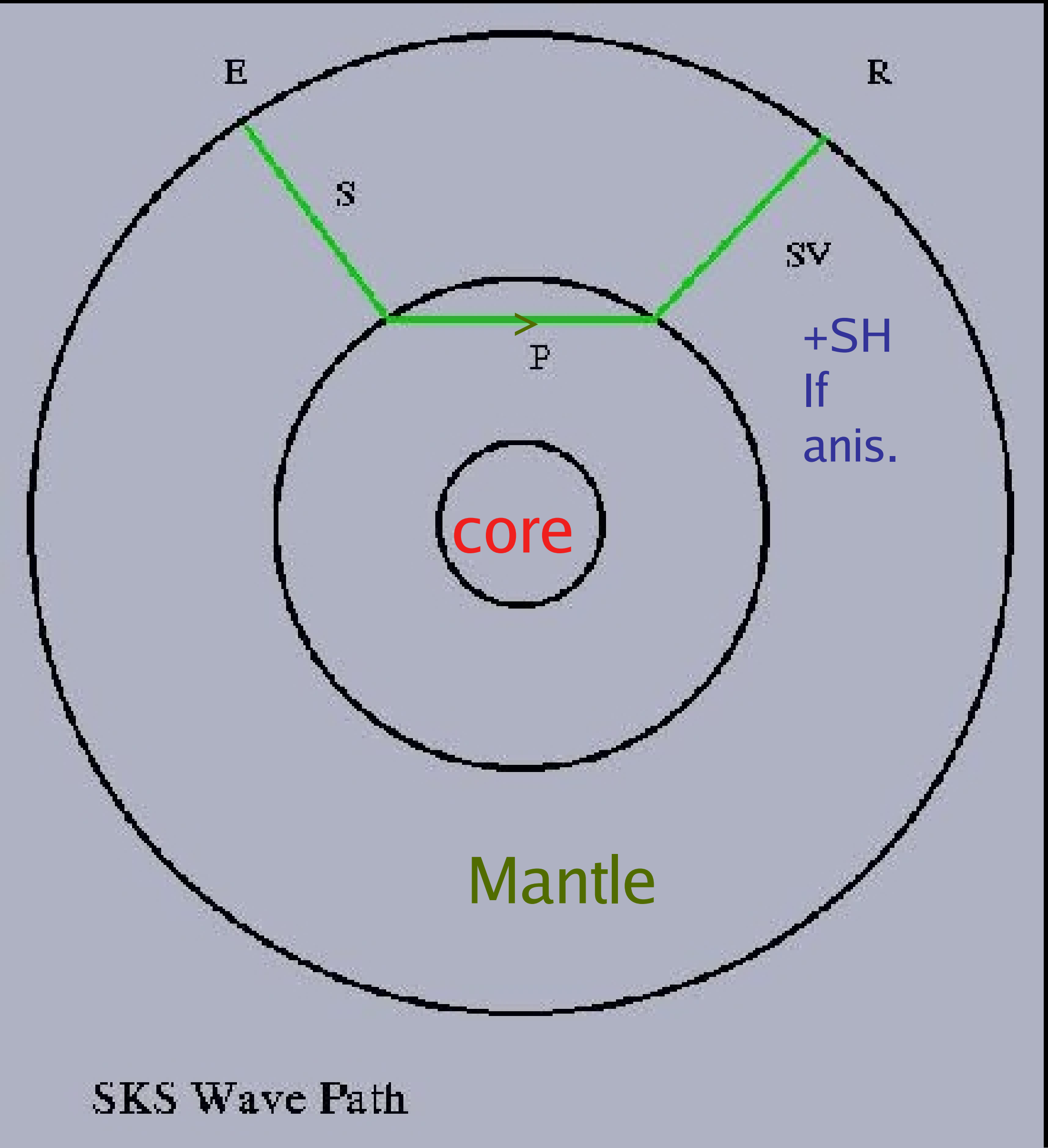


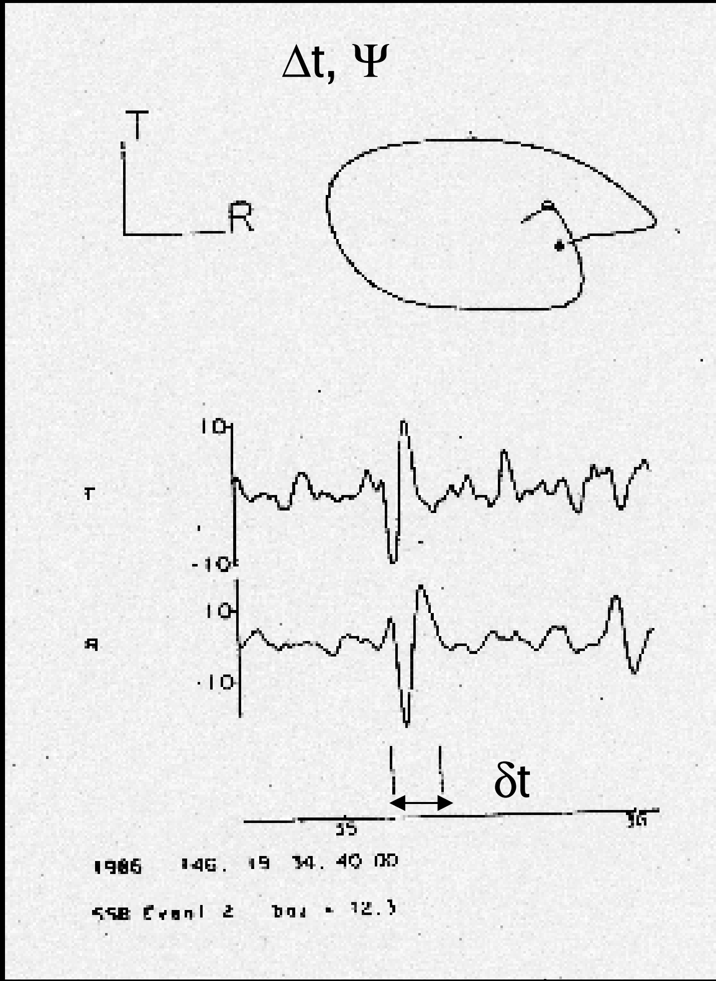
Shear Wave Splitting (Birefringence)



Animation courtesy of Ed Garnero

SKS- Splitting





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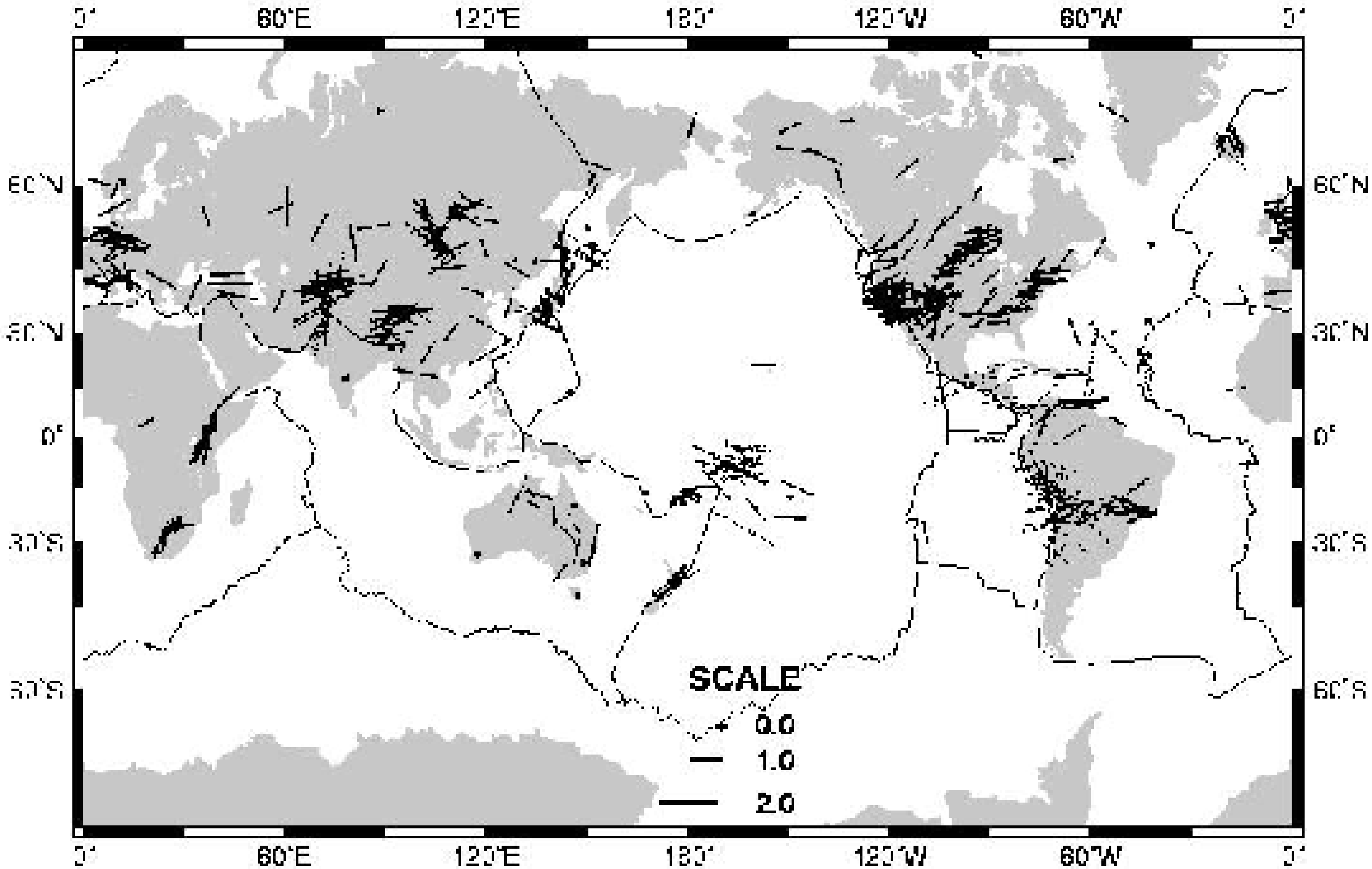
Vinnik et al., 1989

Compilation of S-wave splitting measurements



Savage, Rev. Geophys., 1999

Importance of seismic anisotropy
ANISOTROPY is the Rule not the Exception

Anisotropy is present at all scales

- from microscopic scale to macroscopic scale
-Efficient mechanisms of alignment
(L.P.O.: lattice preferred orientation
S.P.O.: shape preferred orientation; fine layering)

Anisotropy is observed on different kinds of seismic waves
-Body waves (Pn; S-wave splitting)
-Surface waves (Rayleigh-Love discrepancy, azimuthal anisotropy)

ANISOTROPY REFLECTS AN INNER ORGANIZATION

ANISOTROPY IS NOT A SECOND ORDER EFFECT

Effect of anisotropy on surface waves

Effect on eigenfrequency for multiplet k={n,l,m}

$$\frac{\delta\omega_k}{\omega_k} = \frac{\int_\Omega \varepsilon_{ij}^* \, \delta C_{ijkl} \, \varepsilon_{kl} \, d\Omega}{\int_\Omega \rho_0^* \, u_r^* \, u_r \, d\Omega} = \left.\frac{\delta V}{V}\right|_k$$

ε strain tensor, u displacement, δC_{ijkl} elastic tensor perturbation

Phase velocity pertubation $V(T,\theta,\phi,\Psi)$ at point r (θ,ϕ) (Smith&Dahlen, 1973)

$$\delta V(T,\theta,\phi,\Psi)/_V = \alpha_0(T,\theta,\phi) + \alpha_1(T,\theta,\phi)\cos 2\Psi + \alpha_2(T,\theta,\phi)\sin 2\Psi$$
$$+ \alpha_3(T,\theta,\phi)\cos 4\Psi + \alpha_4(T,\theta,\phi)\sin 4\Psi$$

Ψ Azimuth (angle between North and wave vector)

13 parameters

- Best Resolved parameters for Surface Waves

L $= \rho V_{SV}^2$ Isotropic part of V_{SV}.

$\xi = \frac{N}{L} = \frac{V_{SH}^2}{V_{SV}^2}$ Radial Anisotropy.

G, Ψ_G Azimuthal Anisotropy of V_{SV}, also related to SKS splitting (when horizontal symmetry axis).

+ a priori information (from mineralogy, …)

- Body Waves (*Crampin*, 1984)

$\rho V_{qSV}^2 = L + G_c cos2\Psi + G_s sin2\Psi$

$\rho V_{qSH}^2 = N - E_c cos4\Psi - E_s sin4\Psi$

Geodynamic Interpretation

Convective cell: anisotropic parameters

S- Velocity

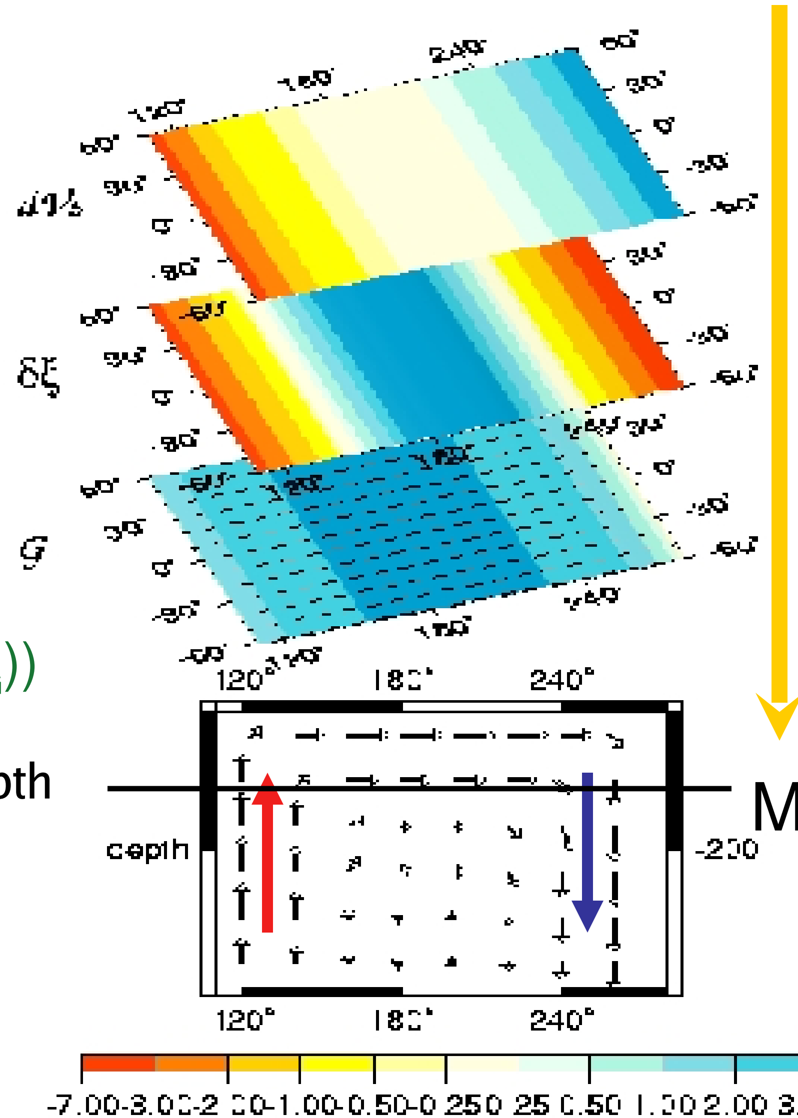
Radial Anisotropy

$\xi = (V_{SH}^2 - V_{SV}^2)/V_{SV}^2$

Azimuthal Anisotropy

$V_{SV} \approx V_{SV0} + {}^{1}/_{2} G\cos(2(\Psi - \Psi_G))$

At a given depth



Interpretation (L.P.O.)

Map Flow

2 D tomography: N cells

Isotropic Inversion:

N independent parameters

0-Ψ term

VR1

Variance reduction

Anisotropic inversion

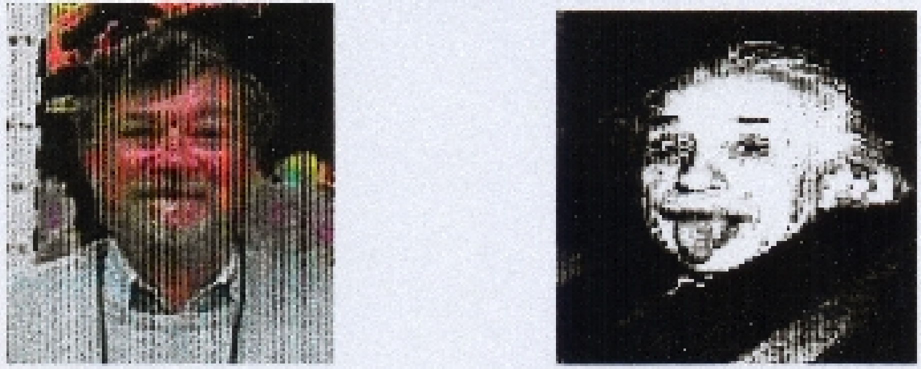
3N' = N

0+2 Ψ term

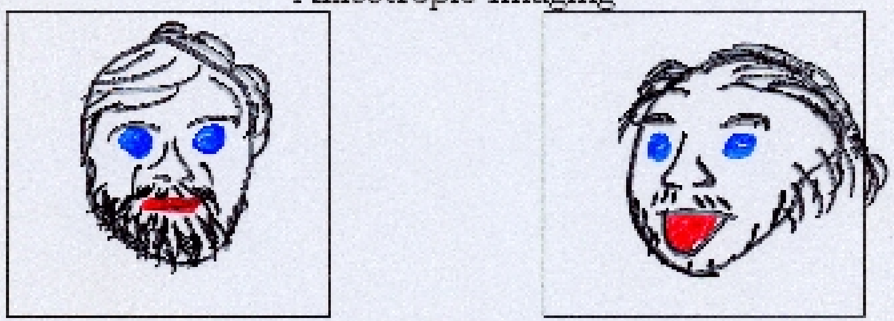
VR2

VR2>VR1 => the anisotropic model can be simpler than the isotropic model

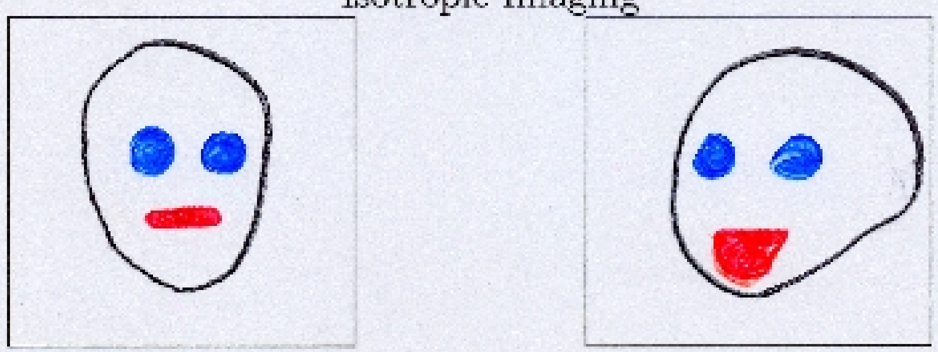
IMAGING OF FAMOUS SCIENTISTS

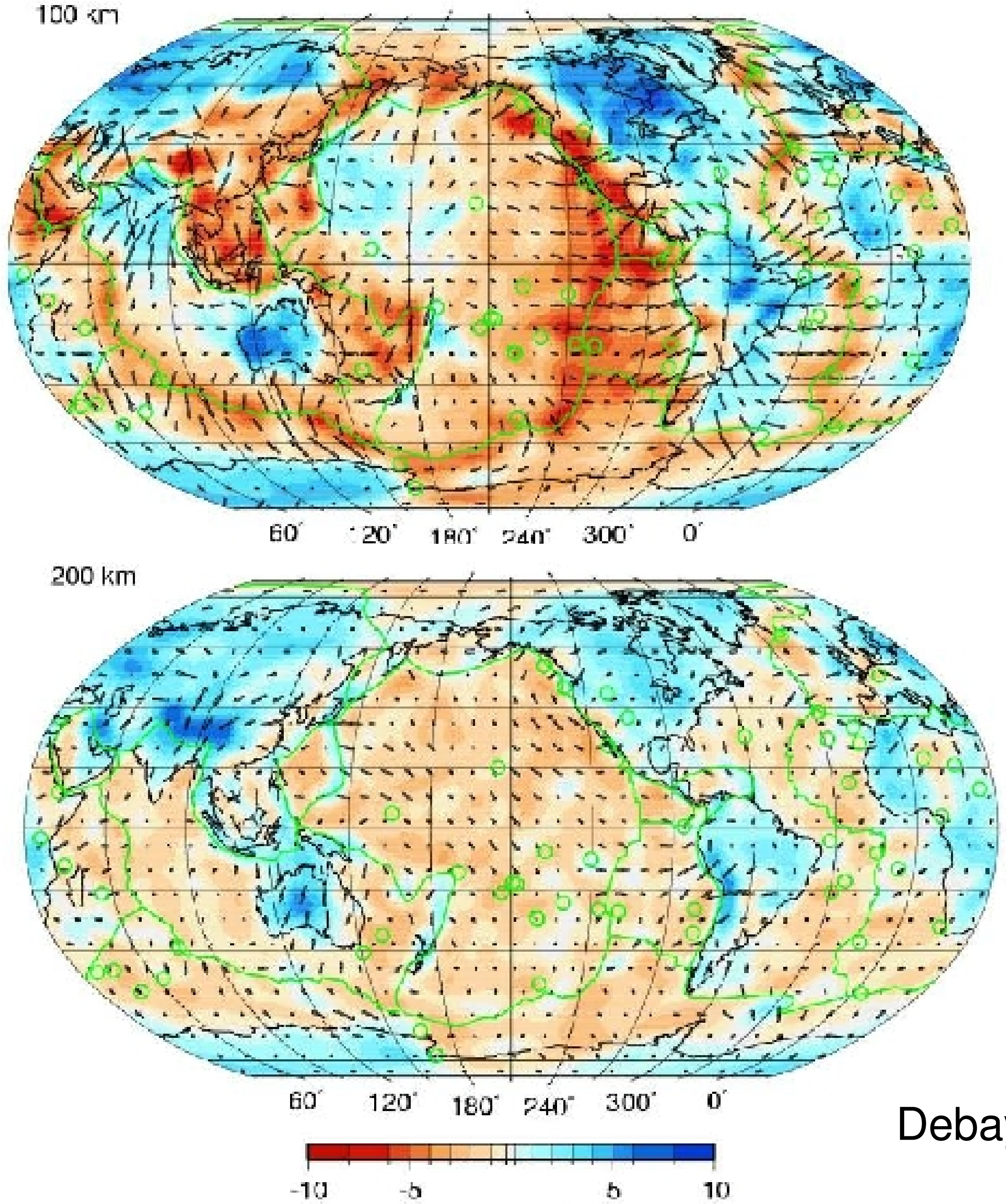


Anisotropic Imaging



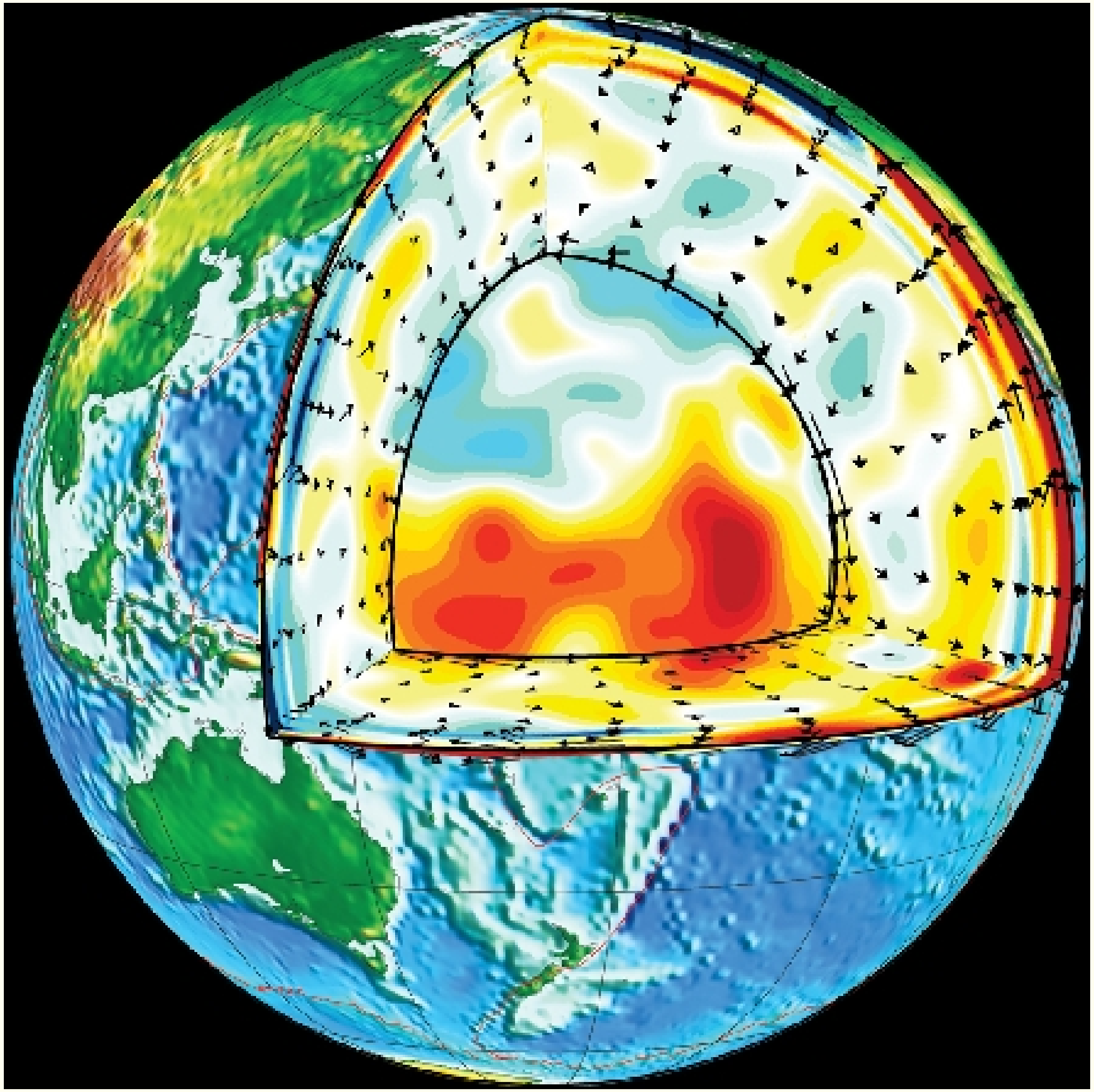
Isotropic Imaging





Debayle et al., 2005

Anisotropy- Geodynamics Relationship



Gaboret et al., 2003

CONCLUSIONS

- Progrès en instrumentation (Fond de Mer, Planète Mars; Spatial)
- Théorie des rais - Modes Normaux -> Méthodes numériques de plus en plus puissantes et précises grace a des ordinateurs de plus en plus puissants.
- Du Global vers le régional- Incorporation de nouveaux paramètres (anisotropie, anélasticité) en tomographie
- Approche pluridisciplinaire systématique: Confrontation des résultats sismologiques, avec expériences analogiques et numériques